Participant Guide

Mathematical Argumentation in Coordinate Geometry

Summer 2007 Workshop

ACCESS MATHEMATICS

Center for Technology in Learning

SCHEDULE

Day 1 "Plane" Old Geometry	Research tasks and information
	Lunch
	A1. Mathematical argumentation in coordinate geometry
	A2. Similar figures: How & why?
	A3. Definition of similarity: What matters?
	A4. Finding similar figures (Minicase 1)
	Daily evaluation
Day 2 "Plane + Algebra = Coordinate Geometry"	B1. Location, location, location
	B2. Exploring rectangles
	B3. Introduction to Geometer's Sketchpad
	B4. Problem solving with rectangles
	Lunch
	B5. Lines of a rectangle
	B6. Equations
	B7. Reading discussion
	B8. Standards & tests (1)
	Daily evaluation
Day 3 Similarity & Congruence	C1. Exploring rectangles (Minicase 2)
	C2. Patterns among rectangles C3. Find similar rectangles
	C4. Standards & tests (2)
	Lunch
	C5. Dilation investigation
	C6. Similarity (Minicase 3) C7. More similarity (Minicase 4)
	Daily evaluation
Day 4 Conversely?	D1. Converse and more
	D2. Converse activity: True or not
	D3. Diagonals (Minicase 5)
	D4. Squares (Minicase 6)
	D5. Standards & tests (3)
	Lunch
	D5. Breaking out of RPA world
	D6. Unpacking
	D7. Taking CG to the classroom
	Wrap-up
	Final evaluation

DAY 1

"PLANE OLD" GEOMETRY

Research tasks and information	
Lunch	
A1. Mathematical argumentation in coordinate geometry	
A2. Similar figures: How & why?	
A3. Definitions of Similarity: What matters?	
A4. Finding similar figures (Minicase 1)	
A5. Daily evaluation	

Note: Homework reading assigned today

A1. Mathematical Argumentation in Coordinate Geometry

During the introduction to the week, note the answers to these questions, and questions you have right now.

1. What will we be doing this week?

2. What is argumentation?

3. Why is mathematical argumentation important?

4. What questions do you still have?

A2. Similar Figures: How & Why?

Use the set of geometric shapes provided. Use any and all of your current knowledge of similarity.

1. Find the similar figures. How do you know they are similar?

2. Arrange figures on a poster to show how you decided which were similar and which were not.

3. Look at other posters. Do you agree with their work? Discuss.

A3. Definition of Similarity: What Matters?

1. Write a definition of similar figures and explain why it is a mathematically good definition.

2. Refine your definition based on discussion.

3. Find and evaluate 2 or 3 textbook definitions. Which definitions hold up?

A4. Finding similar figures (Minicase 1)

1. Read the following minicase & analyze the students' responses.

Ms. Carter asked students to make a set of similar rectangles and explain what they did.
Jose: I kept adding one to the length and the width to get another rectangle. They look alike, so they are similar.
Peter: I think Jose made similar rectangles, because they are all the same shape but not necessarily the same size.

2. Pair-answer. What are Jose's and Peter's arguments? (Consider: what they must think similar is, in order for their thinking to make sense to them.)

3. Pair-collaboration. Draw different ways in which Jose could have drawn the rectangles.

DAY 2

"PLANE + ALGEBRA = COORDINATE GEOMETRY"

B1. Location, location, location	
B2. Exploring rectangles	
B3. Introduction to Geometer's Sketchpad	
B4. Problem solving with rectangles	
Lunch	
B5. Lines of a Rectangle	
B6. Equations	
B7. Reading Discussion	
B8. Standards & tests (1)	
Daily evaluation	

B1. Location, location, location

Exploration 1

1. Sit with a partner so that you cannot see each other's work.

2. On a piece of blank paper, draw any line segment.

3. One person describes how to duplicate his/her drawing. The other follows instructions.

   Rules:
   Explain as if your partner did not understand math conventions.
   Do not use conventional units (i.e., no inches, centimeters, no axes, etc.)

4. When you are done, compare your drawings:
   a. Are your lines congruent and placed in the same location on each page?

   b. How do you know?

Exploration 2

1. Do the same thing, using grid paper.

2. When you are done, compare your drawings:
   a. Are your lines congruent and in the same place?

   b. How do you know?

B2. Exploring rectangles

Exploration 1

1. On a piece of graph paper, draw coordinate axes x and y (horizontal and vertical) so that the origin is (0, 0) at an "interesting place". Scale and label.

2. Draw two rectangles
   Rectangle A – aligned with the grid
   Rectangle B – not aligned with the grid (askew)

3. Explain how you know that A and B are rectangles.

Exploration 2

This time use only rectangles that are aligned with the grid. You're in RPA World

1. On a new piece of graph paper, draw coordinate axes and draw four new rectangles of different sizes and locations. Label the vertices of the rectangles with coordinates.

2. What do you notice about these rectangles and their coordinates? List your observations. Look for patterns.

RPA World–Aligned with the grid:

To make things easier for students, we will often deal only with rectangles that have sides parallel to the axes. We call this RPA World for short.

We often ask you as a teacher to do additional work in Non-RPA World, so that you know its limits.

B3. Introduction to Geometer's Sketchpad

There are many ways of making rectangles in Geometer's Sketchpad (GSP). In this activity, we will create rectangles using the "plot point" feature of GSP.

1. Open GSP

2. Go to graph on the menu and select "define coordinate system"

3. Choose your first point: Click "graph" on the menu and select "plot points."

4. One by one, place three other points to form a rectangle. Pay attention to how you chose them.

5. Connect the points to form a rectangle: Click on two points, then select "construct" and "segment" from the menu.

6. When were you free to choose points? When were you forced into a point? Explain.

B4. Problem solving with rectangles

Try to avoid plotting points while reasoning these through, but then use GSP to check your ideas.

1. Decide which sets of points are vertices of a rectangle.

a.	(1, 4)	(3, 4)	(1, 10)	(3, 10)
b.	(-1, -1)	(-1, 4)	(5, -1)	(5, 4)
c.	(1, 1)	(2, 2)	(3, 3)	(4, 4)
d.	(-2, 4)	(- 6, 4)	(-2, -2)	(-2, -2)
e.	(1, 1)	(-1, 1)	(-1, -1)	(1, -1)

2. In RPA world, which pairs of points provide enough information to determine a single rectangle?
   a. (3, 5), (-4, -2)
   b. (-1, 2), (5, 2)
   c. (0, 0), (3, 9)
   d. (5, 5), (7, 7)
   e. (2, 5), (3, 5)
   f. (3.1, -2.1), (3.1, -3.1)

3. In coordinate geometry, which set of coordinates determines a rectangle?

   a. (-1, 0), (2, 5), (3, 4), (-2, 1)

   b. (1, 6), (2, 8), (3, 2), (4, 4)

B5. Lines of a Rectangle

Exploration 1

1. Use a few rectangles you have drawn in RPA World and draw the diagonals of each.

2. Find the length of each line segment in each rectangle.
   a. How did you do it?
   b. What patterns do you see between the lengths of line segments in each rectangle?

Exploration 2

1. Find the slopes of the diagonals of each rectangle.
   a. What patterns do you see?
   b. What is slope, anyway?

2. Use what you found in 1 to explore the slopes of the sides of your rectangles.
   a. Generalize: What are the slopes of the sides of rectangles in RPA World? How do you know?

B6. Equations

Exploration 1

1. Open B6-equations1.gsp. Follow the instructions. Observe changes in the coefficient of x (call it k). Explain the pattern of changes you observe.

2. Open B6-equations2.gsp. Follow the instructions. Observe changes in the constant (call it b). Explain the pattern of changes you observe.

3. What's the definition of a line that you use when not in coordinate geometry? Use what you found out in 1 and 2 (above) to develop a definition of line in coordinate geometry.

Exploration 2

1. Open B6-equations3.gsp.

2. Display the equations for the lines by highlighting each and choosing Equation in the Measure menu.

3. Play around with dragging and find patterns in the changing equations. Explain the patterns of changes you observe.

B7. Reading discussion

1. **Review** the reading individually and highlight a sentence, phrase, or paragraph that stands out as important. (2 min)

2. **Pair-Answer** your highlighted text and tell your partner why you chose that text. Allow each participant to share uninterrupted for 3 minutes before responding. (10 min)

3. Gather in **small groups** and take turns giving comments using the following prompts. Don't discuss them yet.

   What surprised me . . .
   This article confirmed my understanding . . .
   This article conflicted with my understanding . . .
   As I considered the implications for my classroom . . .
   This article raised questions in my mind . . .
   What was most interesting for me . . .

   Now go ahead and discuss your comments with each other. (15 min)

B8. Standards & tests (1)

1. For each set of standards, summarize what middle schoolers are supposed to learn about coordinate geometry.

2. Note any significant differences between the standards (i.e., NCTM vs CA, vs Oakland/WCCUSD).

3. Compare the coordinate geometry standards with topics you usually teach.

NCTM

Grades 3-5

G.6 Describe location and movement using common language and geometric vocabulary

G.7 Make and use coordinate systems to specify locations and to describe paths

G.8 Find the distance between points along horizontal and vertical lines of a coordinate system

Grades 6-8

G.1 Precisely describe, classify, and understand relationship among types of two- and three-dimensional objects using their defining properties

G.4 Use coordinate geometry to represent and examine the properties of geometric shapes

G.5 Use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides

CA Standards

Grade 7

Algebra and Functions in Grade 7

3.3. Graph linear functions, noting that the vertical change (change in y- value) per unit of horizontal change (change in x- value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Measurement and Geometry in Grade 7

3.2. Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

3.3. Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

Algebra in Grades 8-12

8.0. Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

Geometry in Grade 8-12

4.0. Students prove basic theorems involving congruence and similarity.

15.0. Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles

17.0. Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

Oakland USD Standards

Grade 6

Algebra and Functions

3. Students investigate geometric patterns and describe them algebraically.

3.1. use variables in expressions describing geometric quantities

3.2. express simple relationships arising from geometry in symbolic form.

Grade 7

Algebra and Functions

3. Students graph and interpret linear and some non-linear functions

3.3. graph linear functions, noting that the vertical change (change in y-value) per unit horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Measurement and Geometry

3. Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures

3.2. understand and use coordinate graphs to plot simple figures, determine lengths and areas related tot hem, and determine their image under translations and reflections

3.3. know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and lengths of other segments, and, in some situations, empirically verify the Pythagorean Theorem by direct measurement

DAY 3

SIMILARITY AND CONGRUENCE

IN THE COORDINATE PLANE

C1. Exploring rectangles (Minicase 2)	
C2. Patterns among rectangles	
C3. Find similar rectangles	
C4. Standards & tests (2)	
Lunch	
C5. Dilation investigation	
C6. Similarity (Minicase 3)	
C7. More similarity (Minicase 4)	
Daily evaluation	

C1. Exploring rectangles (Minicase 2)

1. Read minicase & make some notes on your thinking

Students are given a task to create rectangles on the coordinate plane.

Most students draw a rectangle like Figure 1.

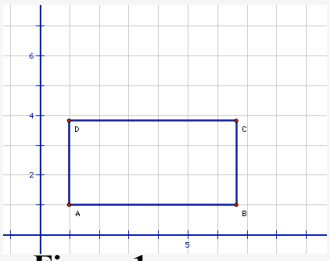


Figure 1

Chris draws a figure differently (Figure 2).

Josh: I think Chris's is a rectangle 'cause it just looks like others.

Zoe: I don't think so. It looks different and it is even tilted.

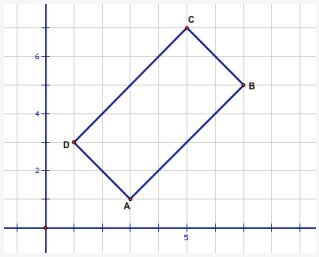


Figure 2

2. Pair-answer. What are Josh and Zoe's working definitions of rectangle?

3. Pair-collaboration. Mathematically, how do we really know if the two are rectangles?

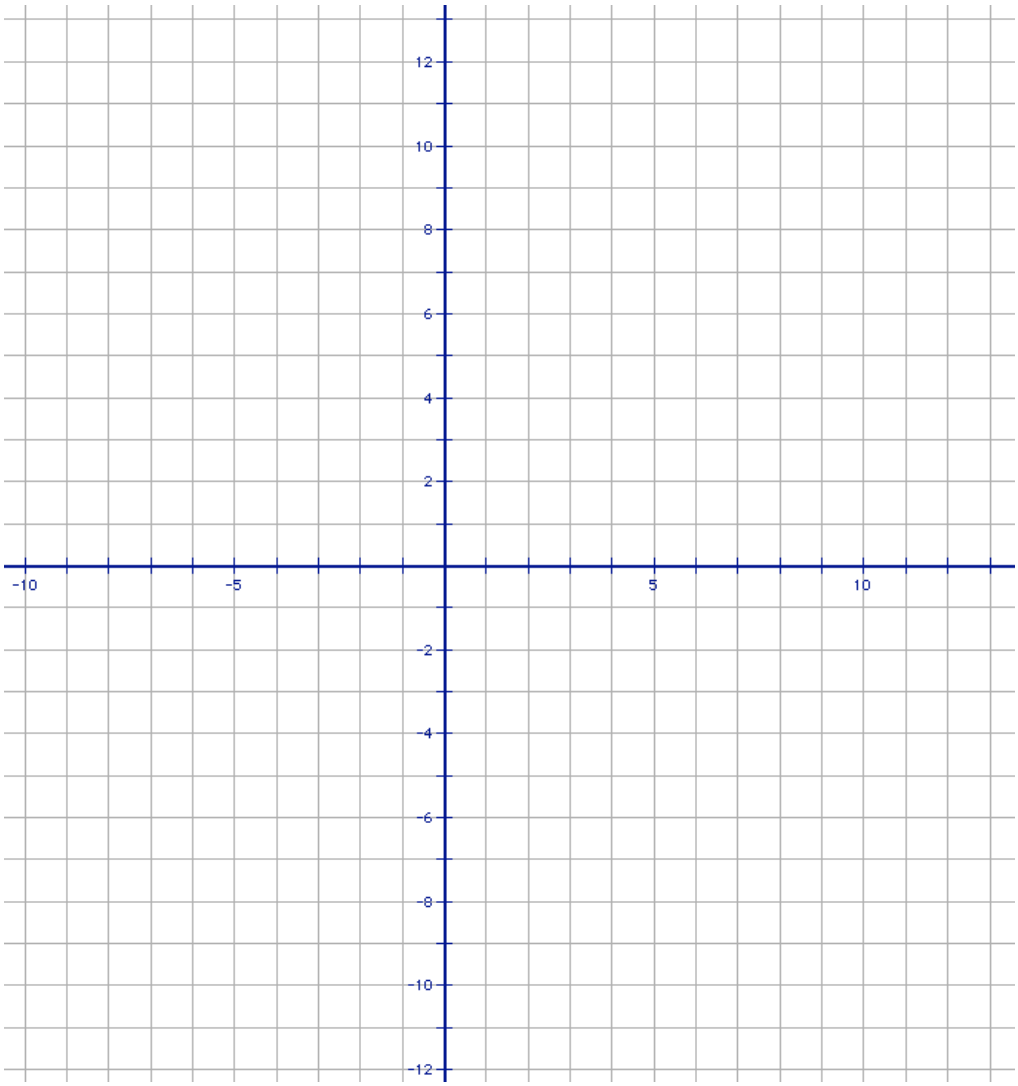
C2. Patterns among rectangles

1. Use the graph paper you are given. Create 5 rectangles so that their diagonal lines coincide with line graphed on your paper.

2. Find relationships among rectangles created.

C3. Find similar rectangles

1. Draw the rectangle (-1, -1), (3, -1), (3, 1), (-1, 1).



2. Try different operations on the coordinates of the rectangle to create a new rectangle. (For example, add 3 to each coordinate or add 3 to the y coordinate only.)

3. Which operations produce similar rectangles? Why?

C4. Standards & tests (2)

1. For each set of standards, summarize what middle schoolers are supposed to learn about coordinate geometry.

2. Note any significant differences between the standards (i.e., NCTM vs CA, vs Oakland/WCCUSD).

3. Compare the coordinate geometry standards with topics you usually teach.

NCTM

Grades 3-5

G.6 Describe location and movement using common language and geometric vocabulary

Grades 6-8

G.3 Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship

G.4 Use coordinate geometry to represent and examine the properties of geometric shapes

G.6 Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling

G.7 Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

NCTM Focal Points

Grade 7

Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).

Connections to the Focal Points

Measurement and Geometry: Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply

proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.

CA Standards

Grade 7

Algebra and Functions in Grade 7

3.3. Graph linear functions, noting that the vertical change (change in y- value) per unit of horizontal change (change in x- value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Measurement and Geometry in Grade 7

3.2. Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

3.4. Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

Algebra in Grades 8-12

8.0. Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

Geometry in Grade 8-12

4.0. Students prove basic theorems involving congruence and similarity.

17.0. Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.

Oakland USD Standards

Grade 6

Algebra and Functions

2. Students analyze and use tables, graphs and rules to solve problems involving rates and proportions.

3. Students investigate geometric patterns and describe them algebraically.

3.1. use variables in expressions describing geometric quantities

3.2. express simple relationships arising from geometry in symbolic form.

Grade 7

Algebra and Functions

3. Students graph and interpret linear and some non-linear functions

3.3. graph linear functions, noting that the vertical change (change in y-value) per unit horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Measurement and Geometry

3.2. understand and use coordinate graphs to plot simple figures, determine lengths and areas related tot hem, and determine their image under translations and reflections

3.4. demonstrate an understanding of when two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

C5. Dilation investigation

The Glossary for the California Mathematics Content Standards defines similarity in terms of dilations. The definition is explained in coordinate geometry terms. Let's see what we can make of it, using GSP and all we have learned.

[PS. This is NOT a student activity.]

1. Using the GSP file "C5-DilationExploration.gsp," to analyze the definitions below. (http://www.cde.ca.gov/be/st/ss/mthglossary.asp).

   Similarity.
   In geometry, two shapes R and S are similar if there is a dilation D (see the definition of dilation) that takes S to a shape congruent to R. It follows that R and S are similar if they are congruent after one of them is expanded or shrunk.

   Dilation.
   In geometry, a transformation D of the plane or space is a dilation at a point P if it takes P to itself, preserves angles, multiplies distances from P by a positive real number r, and takes every ray through P onto itself. In case P is the origin for a Cartesian coordinate system in the plane, then the dilation D maps the point (x, y) to the point (rx, ry).

   GSP Hints:
   - Increase and decrease scale factor, and see how the figure changes.
   - Double click on the scale factor and plug in a number.
   - Explore coordinates, length ratio, and area ratio.
   - Try with different scale factors and explore coordinates, length ratio, and area ratio.
   - Move X away from the origin.

2. Write an explanation or write a new definition in terms meaningful to you and your partner.

C6. Similarity (Minicase 3)

1. Read the following minicase.

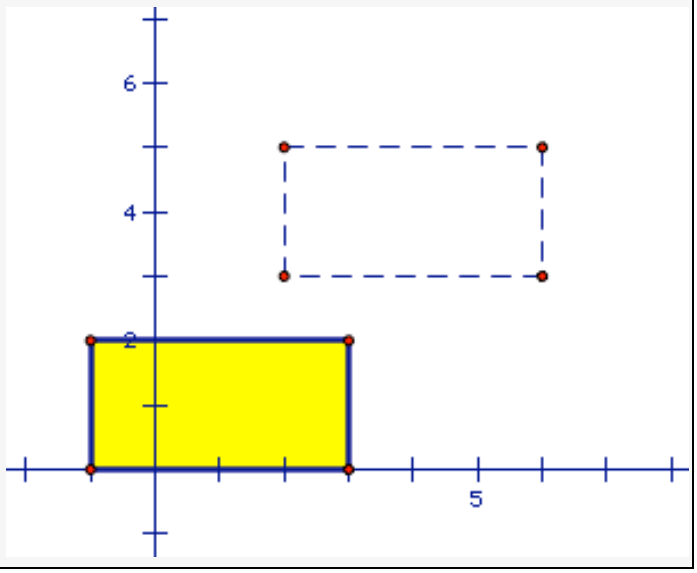
Students try to come up with a strategy to create similar rectangles given the rectangle (-1, 2), (-1, 0), (3, 2), (3, 0).

Kevin: Let's add 3 to everything.

Selma: No you can't. You can't add.

Kevin: Why not? Let's just add and see how it looks. <Kevin adds 3 to each coordinates and got (2, 5), (2, 3), (6, 5), (6, 3). Look. They look the same.

Selma: You are not supposed to add... But they look similar. That's weird. [Selma stares at the two rectangles] Wait. You just made the same thing. You have to make a SIMILAR.

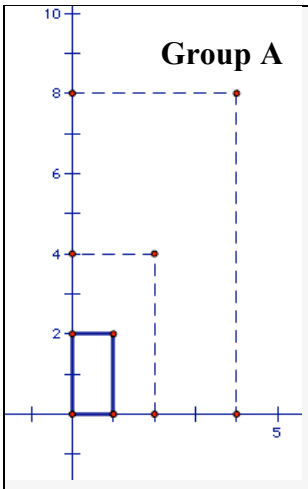


2. Pair-answer: Explain in your own words what Kevin's working definition of similarity is. Have your partner explain Selma's argument.

3. Pair-collaboration. Determine whether the new figure (Kevin's) is similar.

C7. More similarity (Minicase 4)

1. Read the following minicase and summarize the different arguments.

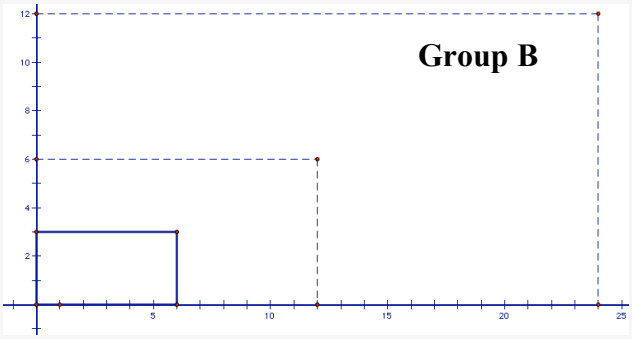


Students are creating similar rectangles in groups.

Group A started with a rectangle, (0, 0), (1, 0), (0, 2), (1, 2) and kept multiplying 2 to each coordinates to create other rectangles, (0, 0), (2, 0), (0, 4), (2, 4), and (0, 0), (4, 0), (0, 8), (4, 8), and so forth.

Group B started with a rectangle, (0, 0), (6, 0), (0, 3), (6, 3) and kept multiplying 2 to each coordinates to create other rectangles to the original, (0, 0), (12, 0), (0, 6), (12, 6), and (0, 0), (24, 0), (0, 12), (24, 12), and so forth.

After the activity, the whole class discusses the results.



Kim: Ms. Apple, I think group A and group B used the same strategy. Doesn't that mean that theirs are all similar to each other?

Monica: <from group A> No they are not. Ours look more like tall and skinnier but theirs look sort of flat and fat <referring group B>.

David: <from group B> I think Monica is right. Our diagonal line doesn't match theirs. So ours are different from group A's.

Kim: Different doesn't necessarily mean they are NOT similar. Sizes don't need to be the same, remember?

2. Team-answer. What are Kim, Monica and David's arguments? Represent one of the three student's arguments and explain to your teammates what you think the student is saying. Listen to your teammates as they describe the other students' arguments.
3. Team-collaboration. Create example(s) that can help Kim, Monica and David refine or refute their conjectures.

DAY 4

CONVERSELY?

D1. Converse and more	
D2. Converse sctivity: True or not	
D3. Diagonals (Minicase 5)	
D4. Squares (Minicase 6)	
D5. Standards & tests (3)	
Lunch	
D6. Breaking out of RPA world	
D7. Unpacking	
D8. Taking CG to the classroom	
D9. Wrap-up	
Final evaluation	

D1. Converse and more

Let's assume we know this much is true:
If it's raining, the window is wet.

What else do we know, based on that true statement?

Consider the possibilities:

If the window is wet, it is raining.
Does it have to be true? Explain.
If not, draw a situation in which it is false. →

If it is not raining, then the window is not wet.
Does it have to be true? Explain.
If not, draw a situation in which it is false. →

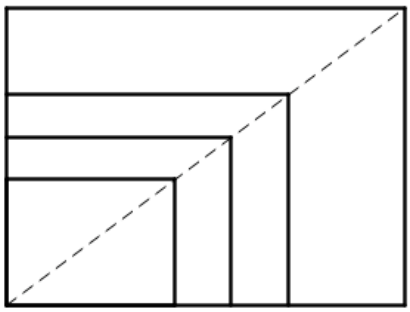
If it the window is not wet, then it's not raining.
Does it have to be true? Explain.
If not, draw a situation in which it is false. →

D2. Converse activity: True or not

First decide which of these statements are true. Then consider the converse: is it also true? Why or why not?

1. If two figures are similar by a scale factor of n, then the ratio of the areas is 1: n^2

2. In coordinate geometry, if the rectangles in a set are all similar, then their corresponding diagonals will fall on one line.



3. All squares are rectangles.

4. If two shapes are congruent, then they are similar.

5. Any statement that is true about squares is also true about rectangles.

6. If it is a right triangle, then $a^2+b^2=c^2$, where c is the hypotenuse.

7. If 2 lines are parallel, then they are equidistant everywhere.

D3. Diagonals (Minicase 5)

1. Read the following minicase and write down some thoughts.

Students made five similar rectangles in groups.

One group made their rectangles by multiplying each side of the original by 2, again and again. They found an interesting pattern in the rectangles.

Amy: Look! We found that diagonal lines of all rectangles we made fall onto one line.

Tom: <from another group> Wow. That's neat. Let's try our rectangles.

Tom and his group members tried with the rectangles they made and found that their rectangles also shared one diagonal line.

Tom: See? Our rectangles are similar.

Teacher: Why?

Tom: Because they share a diagonal line.

2. Pair-answer. Tell your partner whether Tom's argument is convincing.

3. Pair-collaboration. Discuss with your partner: What does converse have to do with this case?

D4. Squares (Minicase 6)

1. Read the following minicase and write down some thoughts.

Watanabe created five squares: 1x1, 2x2, 3x3, 4x4, 5x5.

Watanabe: I made them by adding 1 to the length and the width. These are all similar.

Jesse: Okay then I will make similar rectangles by doing the same thing that Watanabe did: 1x2, 2x3, 3x4, 4x5, 5x6. That works, right?

2. Pair-answer. Present one of the student's working conjecture to your partner. Listen while she/he does the same with the other student.

3. Pair-collaboration. Discuss with your partner: Is Wantanabe's conjecture always true? Is Jesse's? Explain.

D5. Standards & tests (3)

1. For each set of standards, summarize what middle schoolers are supposed to learn about coordinate geometry.

2. Note any significant differences between the standards (i.e., NCTM vs CA, vs Oakland/WCCUSD).

3. Compare the coordinate geometry standards with topics you usually teach.

NCTM

Grades 6-8

G.3 Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship

K-12

P.5 Recognize reasoning and proof as fundamental aspects of mathematics;
P.6 Make and investigate mathematical conjectures;
P.7 Develop and evaluate mathematical arguments and proofs;
P.8 Select and use various types of reasoning and methods of proof.
P.9 Organize and consolidate their mathematical thinking through communication;
P.10 Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
P.11 Analyze and evaluate the mathematical thinking and strategies of others;
P.12 Use the language of mathematics to express mathematical ideas precisely.
P.14 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
P.16 Create and use representations to organize, record, and communicate mathematical ideas;
P.18 Use representations to model and interpret physical, social, and mathematical phenomena.

California

Geometry

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.
2.0 Students write geometric proofs, including proofs by contradiction.
3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.
22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.

D6. Breaking out of RPA world

1. What did we find out about RPA World? List all conclusions.

2. Which of these are also always true in non-RPA world?

Conjecture

Someone makes a mathematical statement, best guess, but we don't know for sure

Justification

Together, we figure out: is it true or not?

Conclusion

We agree: it is true

OR

We don't agree: no, it's really this way...(new conjecture)

STANDARDS

NCTM

Grades 3-5

G.6 Describe location and movement using common language and geometric vocabulary

G.7 Make and use coordinate systems to specify locations and to describe paths

G.8 Find the distance between points along horizontal and vertical lines of a coordinate system

Grades 6-8

G.1 Precisely describe, classify, and understand relationship among types of two- and three-dimensional objects using their defining properties

G.3 Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship

G.4 Use coordinate geometry to represent and examine the properties of geometric shapes

G.5 Use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides

G.6 Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling

G.7 Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

K-12

P.1 Build new mathematical knowledge through problem solving;

P.2 Solve problems that arise in mathematics and in other contexts;

P.3 Apply and adapt a variety of appropriate strategies to solve problems;

P.4 Monitor and reflect on the process of mathematical problem solving.

P.5 Recognize reasoning and proof as fundamental aspects of mathematics;

P.6 Make and investigate mathematical conjectures;

P.7 Develop and evaluate mathematical arguments and proofs;

P.8 Select and use various types of reasoning and methods of proof.

P.9 Organize and consolidate their mathematical thinking through communication;

P.10 Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

P.11 Analyze and evaluate the mathematical thinking and strategies of others;

P.12 Use the language of mathematics to express mathematical ideas precisely.
P.13 Recognize and use connections among mathematical ideas;
P.14 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
P.15 Recognize and apply mathematics in contexts outside of mathematics.
P.16 Create and use representations to organize, record, and communicate mathematical ideas;
P.17 Select, apply, and translate among mathematical representations to solve problems;
P.18 Use representations to model and interpret physical, social, and mathematical phenomena.

NCTM Focal Points

Grade 7

Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).

Connections to the Focal Points

Measurement and Geometry: Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.

CA Standards

Grade 7

Algebra and Functions in Grade 7

3.3. Graph linear functions, noting that the vertical change (change in y- value) per unit of horizontal change (change in x- value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Measurement and Geometry in Grade 7

3.2. Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

3.3. Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

3.4. Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

Algebra in Grades 8-12

8.0. Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

Geometry in Grade 8-12

4.0. Students prove basic theorems involving congruence and similarity.

15.0. Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles

17.0. Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

Oakland USD Standards

Grade 6

Algebra and Functions

2. students analyze and use tables, graphs and rules to solve problems involving rates and proportions.

2.3. solve problems involving rates, average speed, distance and time.

3. Students investigate geometric patterns and describe them algebraically.
3.1. use variables in expressions describing geometric quantities
3.2. express simple relationships arising from geometry in symbolic form.

Grade 7

Algebra and Functions

3. Students graph and interpret linear and some non-linear functions
3.3. graph linear functions, noting that the vertical change (change in y-value) per unit horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Measurement and Geometry

3. Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures
3.2. understand and use coordinate graphs to plot simple figures, determine lengths and areas related tot hem, and determine their image under translations and reflections
3.3. know and understand the Pythagorean theorem and use it to find the length of the missing side of a right triangle and lengths of other segments, and, in some situations, empirically verify the Pythagorean theorem by direct measurement
3.4. demonstrate an understanding of when two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

Pacing Guides (OUSD, WCCUSD)

OUSD Math 7

10/9 - 11/17/06	11/27/06 - 1/26/07	1/29 - 3/9/07	3/12 - 4/27/07
Chapter 3: Operations with Integers - 3.1 Integers and Absolute Value - 3.8 The Coordinate Plane	Chapter 8: Geometry Concepts - 8.1 Points, Lines and Planes - 8.3 Parallel and Perpendicular Lines - 8.5 Polygons and Congruence	Chapter 7: Proportional Reasoning - 7.1 Ratios and Rates - 7.2 Writing and Solving Proportions - 7.3 Scale Drawings and Models	Chapter 11: Graphing Linear Equations (& Inequalities) - 11.1 Functions - 11.2 Linear Equations and Linear Functions - 11.3 Graphs of Linear Functions - 11.5 The Slope of a Line - 11.6 The Slope-Intercept Form

WCCUSD Pre-Algebra

1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
8/38 – 20/37/06	10/30-1/19/07	1/23-3/30/07	4/2-6/13/07
- Order of Operations - Properties - Variables & Expressions - Integers Operations - Absolute Value - Equations & Inequalities	- Rules of Exponents - Scientific Notation - Rational Numbers - Operations w/Fractions - Ratios, Proportions, Percents	- Percent of Change - Linear Equations (Graphing, Slope & y-intercept, relationships) - Pythagorean Theorem - Box-and Whisker Plots	- Geometry (Angle relationships, Polygons, Perimeter & area, Surface area, volume) - Data Analysis - Probability