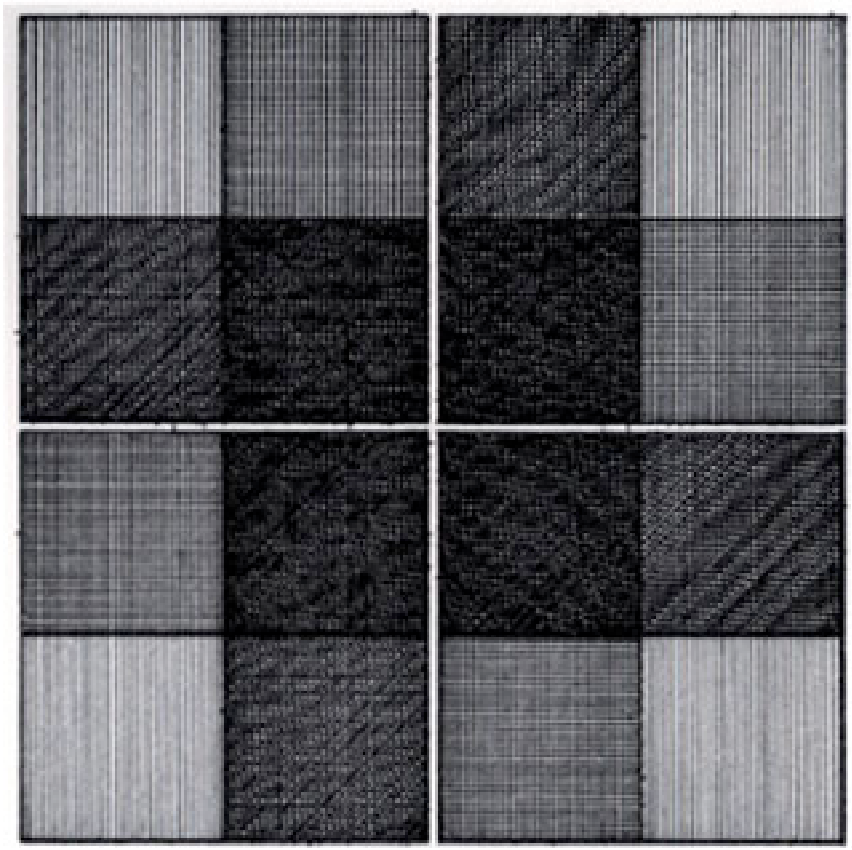
Rectangles in the Coordinate Plane



Teacher Notes

Sol LeWitt, Drawing Series—Composite, Part I-IV, #1-24, A+B(detail), 1969. © Estate of Sol LeWitt/Artists Rights Society (ARS), New York.

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Rectangles in the Coordinate Plane addresses the basics of coordinate geometry, properties of rectangles, and making mathematical arguments using these ideas.

4 lessons (about 45 minutes each)

Day 1: Draw My Line
Day 2: Making Rectangles 1, begin Making Rectangles 2
Day 3: Complete making Rectangles 2
Day 4: Rectangle Strategy and Pick the Rectangles

Standards:

This unit gives students an opportunity to develop strong mathematical reasoning skills as called for in the California Mathematics Standards, grades 6 and 7. This is done in the context of coordinate geometry, addressing 5th through 7th grade Standards in a way that leads directly to high school Geometry.

Materials:

Printouts of student materials in booklet form
Extra graph paper (Some is provided in the student materials)
Rulers, colored pencils
Accompanying PowerPoint presentation and a way to show it to the class

Draw My Line

Materials*:* Graph paper and unlined paper, a couple sheets for each pair of students.

Purpose: Help students see that coordinate representation is a great tool to describe and communicate mathematical objects

Student learning objectives:

- Knowing the necessity and benefits of grids or coordinate representation
- Knowing how to informally communicate mathematical concepts to describe location of a line, such as angles, slopes, lengths, distances, directions, and origin.

Note. Coordinate geometry is all about location. In #1, students should experience some difficulty in communicating the location, but use strategies that suggest the need for graph paper. In #2 students may not make axes— simply using the grid is sufficient.

1. Work with a partner where you cannot see each other's work, but you can hear each other.
    a. On blank unlined paper, draw a line segment—anywhere, any length.
    b. Now tell your partner how to make exactly the same drawing BUT
        - You cannot show your drawing.
        - You cannot use any words for units, like inches or centimeters.
    c. Check to see how well your partner did.
    d. Write down some of the words you used to give instructions.
2. Do the same thing, but this time use the graph paper and switch roles. How did you use the graph lines to help you tell your partner what to draw?

1. (10-15 minutes). Student work in pairs, facing each other with a folder or other visual barrier between them so that they can't see each other's work.

Note. Ask students *in advance* to notice what words they use in communicating with a partner.
Typical language students might use:
about half way in from the middle
close to the edge
start from the bottom left corner
put a point in center and draw a line in the northeast direction
the length of the line is the same as your index finger

Note. Discuss how students used language to indicate distance or direction. Students will often define an "origin".

2. (5-10 minutes). Same format as the previous task. Now students can take advantage of the grids which provide them with a standard unit.

3. (10 minutes). Whole class discussion.

Prompting questions:

1. Which was easier, graph or unlined paper? Why?
2. What difficulties did you have when trying to communicate the location and the length when using the unlined paper?
3. What are the important features of the grid paper that you used?
   Origin and distance or counting or direction are the relevant ones
4. How did those features help you describe the length and the location when using the grid paper?
5. What kind of words did you use in #1 to address those same things?
   Origin: The corner
   Distance and units: about ½ way, the size of the paper is the unit.
   Direction: from the corner

4. (10 minutes). Show the history of coordinate geometry PowerPoint. See notes in the document to guide you on what to say.

Making Rectangles

Use the next page for your work.

1. Draw an x-axis and a y-axis and label the units. Do not put the origin in the middle of the page!

2. Draw two rectangles and label the vertices.
   a. One rectangle—"lined up" with the grid. Its sides are parallel to the axes.
   b. Another rectangle—"at a tilt." Its sides are NOT parallel to the axes.

3. Describe how you drew each rectangle.

Materials: Colored pencils or thin line markers (not required). Straightedges.

Purpose: Help students understand coordinate representation of rectangles through mathematical argumentation.

Student learning objectives:

1. Mathematics
1.1. Knowing how to locate and name a point on a coordinate plane
1.2. Knowing to apply what they already know to a new situation in terms of right angles, parallel and perpendicular lines, and opposite sides having the same length.
1.3. Making connections between plain geometry and coordinate geometry

2. Argumentation
2.1. Clarifying meanings of everyday and mathematical terms
2.2. Generating conjectures about lines, angles, and rectangles
2.3. Justifying conjectures
2.4. Providing counterexamples to refute arguments

1. (5 minutes). Review conventions of coordinate graphing—how to draw and label axes and points.

You can remediate here, if students don't know these basics:
The axes are at right angles.
Every point is labeled with 2 numbers: one for its position relative to the x-axis; one for its position relative to the y-axis.
Each point has a unique pair of coordinates.

Note. We ask students to locate the origin at a point other than the center of the paper to establish or reinforce the idea that the origin can be anywhere. This also enables students to work with a wider variety of coordinates

2. (10 minutes). Students should work individually. Make sure they know what "coordinates of vertices" means. Reinforce basics above as students do their work.

Prompting questions:

1. What is your definition of a rectangle?
2. How do you know for sure what you drew is a rectangle?
3. How did you draw a right angle?

Note. You will need to establish, together, a commonly accepted definition of rectangle—one that they learned before coordinate geometry. Example: a rectangle is a quadrilateral with four right angles. This can be done during the discussion. Students use very basics of coordinate geometry to show that the rectangles they have drawn do "qualify" under that definition.

3. (15 minutes). You may want to give students time to work alone, in pairs or groups before a class discussion.

Prompting questions:

1. What is your definition of a rectangle?
2. What do you mean by "quadrilateral"?
3. Did everyone use the same definition? Does anyone have a different definition?
4. How is your definition is different or similar to Jane'
5. Can you tell me why you think yours is a rectangle?
6. How did you draw a right angle?
7. How do you know it is a right angle?
8. Would you say those two lines are parallel?
9. What do you mean by "parallel"?
10. Are you convinced by what Jane said?
11. Can someone summarize what Jane's argument was?

Note. It is not easy to show that the tilted figure is, in fact, a rectangle. It is fine to stop with that statement of difficulty, and not require full justifications. The next activity focuses on rectangles with sides parallel to the axes, and this activity motivates that restriction.

Making rectangles II

In this activity, use only rectangles that have sides parallel to the axes.

1. Draw an x- and a y-axis and label the units. Do not put the origin in the middle of the page!

2. Draw 4 rectangles in different places on the coordinate grid. Label the vertices.

3. Look at the coordinates of the vertices for each rectangle. Look for patterns. Describe the patterns that you see, using words and symbols.

Materials: Colored pencils or thin line markers (not required). Straightedges.

Purpose: Help students understand properties of rectangles expressed in coordinate geometry through mathematical argumentation.

Student learning objectives:

1. Mathematics. Knowing:
1.1. patterns of coordinates in rectangles overall, and along any side of a rectangle
1.2. how to calculate the length of a side using coordinate pairs
1.3. patterns of coordinate pairs of two endpoints of a diagonal line in a rectangle

2. Argumentation
2.1. Clarifying meanings of everyday and mathematical terms
2.2. Generating conjectures about lines, angles, and rectangles
2.3. Justifying conjectures
2.4. Providing counterexamples to refute arguments

1. (5 minutes). Use this time to again reinforce the basics of coordinate geometry—axes, origin, coordinates.

Note. We ask students to locate the origin at a point other than the center of the paper to establish or reinforce the idea that the origin can be anywhere. This also enables students to work with a wider variety of coordinates

2. (10 minutes). Encourage students to draw very different shapes and sizes of rectangles, located in a variety of places on the paper. Advanced students should make rectangles that cross quadrants.

3. (25 - 40 minutes, can span two days).

- Students can look for and describe patterns in pairs or small groups. Have them spend time individually, considering why the patterns work.
- Then move to small groups, having them present their arguments to each other. Circulate, helping them generalize and clarify the statements of their patterns.

- Then lead a whole class discussion with the aim of arguing for and against general statements about patterns.

Possible patterns students might find:
There are four different numbers in coordinates in a rectangle.
The two coordinate pairs on the horizontal line share y-coordinates.
The two vertices on the line that is parallel to the y-axis have the same x-coordinates.
I can calculate the length of the sides by subtracting coordinates.

Note. Student responses may not be these exact sentences. You need to clarify with questions to figure out what they are trying to say. You may realize that what *you think* they meant can be different from what they actually meant to say.

Prompting questions:
Finding patterns

1. Compare the coordinates of the left sides of two rectangles. What do they have in common?
2. Can you make a table of the coordinates? What does it show?
3. How do the coordinates relate to the lengths of the sides?

Describing patterns

4. What are the patterns you found?
5. What do you mean by "…"?
6. Can you restate what you said as in "If….., then…."?
   For example, "*If* you have a rectangle, *then* its vertices have four different numbers in their coordinates."

Justifying patterns

7. Why does your pattern work?
8. How do you know that it would work all the time? Would it work with other rectangles?
9. Does anyone disagree with what Jane said?
10. Do we all agree with Jane's argument?

Rectangle Strategy

Materials: Colored pencils or thin line markers (not required).

Purpose: Help students apply their knowledge to different situations

Student learning objectives:

1. Mathematics. Knowing:
1.1. to apply patterns that they know to a different situation
1.2. two points that have neither x- nor y-coordinates in common determine a rectangle
1.3. that two such points determine a diagonal line of a rectangle, of coordinates in rectangles overall, and along any side of a rectangle

Use the coordinate grid on the next page to play this game. Use only rectangles that are aligned to the grid.

RULES OF THE GAME
With a partner, take turns making vertices of a rectangle. You can place the points wherever you like, as long as they can be vertices of the same rectangle.

What's the strategy?
How many turns does it take before there is ONLY ONE rectangle you can make?

Rules of the game. (15 minutes). Have students work in pairs and play the game. Suggest they play two or three times to try out different strategies.

Prompting questions:

1. What is your strategy for picking a vertex?
2. How did you come up with your strategy?
3. Why would your strategy work to win?

What's the strategy? (10 minutes).

Prompting questions:
1. Who won? Can you tell me your strategy?
2. What is the pattern in the vertices that you picked in your strategy?
3. How many turns did it take before there is only one rectangle you can make?
4. How did your strategy work to win?
5. Why did your strategy "force" you into one rectangle?

Pick the Rectangles

Purpose: Apply knowledge of patterns of coordinates to predict rectangles.

Student learning objectives:
Students apply what they have learned in other lessons.

1. Before you plot these points, decide which sets of 4 (a, b, c, d and e) can be the vertices of a rectangle and which cannot.

a.	(1,4)	(3,4)	(1,10)	(3,10)
b.	(-1,-1)	(-1,4)	(5,-1)	(5,4)
c.	(1,1)	(2,2)	(3,3)	(4,4)
d.	(-2,4)	(-6,4)	(-2,-2)	(-2,-2)
e.	(1,1)	(-1,1)	(-1,-1)	(1,-1)

2. Plot the points for a-e. Were you right?

Summary. (10 minutes). Ask students to summarize what they have learned.

Elicit these:

- *Review of basics of coordinate geometry –terms and how to plot points*
- *For rectangles "aligned with the grid", the coordinates of their vertices form this pattern: (a,b), (a,d), (c,b), (c,d)*
  (Help students use this algebraic formulation if they have not so far.)
- *How to make mathematical arguments--conjecturing and justifying.*

Apply with 1, 2. (15 minutes).

- Tell students to apply what they have learned to decide if the sets of coordinates a-e can be vertices of a rectangle. Have them explain their answers in writing or orally.
- Remind students about their work in the Making Rectangles activity—there are rectangles, not aligned with the grid, that don't follow the pattern they uncovered (above). But if they follow the pattern, then we know they are rectangles. **Answers: a, b and e are rectangles.**
- Have students plot each set of points to verify their answers in 1. They should resolve wrong answers.