An Implementation Study of the Bridging Professional Development Program

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Abstract

This report presents findings from an implementation study of the Bridging Professional Development Program, which provides in-service teacher professional development (PD) for middle school mathematics teachers to support their facilitation of mathematical argumentation. Mathematical argumentation is a fundamental disciplinary practice, essential to developing mathematical proficiency. The Bridging PD approach integrates the development of mathematical content knowledge with the use of “successive approximations of classroom practice”—strategically sequenced workshop-based simulations—to build “bridges” from PD workshop to classroom practice. The PD model was implemented within a 2-year small-scale randomized experiment impact study. Findings showed growth in teachers’ mathematical knowledge and increased classroom argumentation discourse. Looking closely at teachers’ classroom discourse, we provide narrative of examples of two ways that teachers used their practices to facilitate argumentation—orchestrating participation and Socratic questioning. The Bridging model contributes to the repertoire of pedagogies that mathematics teacher educators can use to enable teachers to make basic shifts from traditional to more discourse-oriented practices.

Mathematics educators, mathematicians, and philosophers agree that mathematical argumentation is necessary for learning mathematics (e.g., Kuhn, 2005; Lakatos, 1976; Lampert, 1990; Romberg, Carpenter, & Kwako, 2005; Thurston, 1998; Yackel, 2001). It is what professional mathematicians do—they make conjectures and justify why they are true or not. It can also be an important way to learn. Engaging in these epistemic practices can provide opportunities to construct conceptually rich understandings and develop intellectual autonomy and confidence in one's mathematical abilities (Yackel & Cobb, 1996). Argumentation has also gotten attention in recent years as Common Core Math Practice Standard 3: "construct viable arguments and critique the reasoning of others" (CCSSM; Common Core State Standards Initiative, 2010).

The Bridging Professional Development Program ("Bridging PD") was designed to support in-service middle school mathematics teachers to facilitate what we call *classroom mathematical argumentation* (CMA). CMA is the whole-class discursive process of argumentation that is collaborative among the teacher and students. It emphasizes not only the content and cognitive aspects of argumentation but also its social aspects. We have chosen to focus on the middle school years because these are a particularly important and vulnerable transition point in the school trajectory, when mathematical concepts become increasingly difficult and abstract (Nathan & Koellner, 2007; Leinhardt, Zaslavsky, & Stein, 1990). This is potentially a high-leverage intervention point at which teacher knowledge and practices can make a significant difference for students.

When the project began in 2004, we sought to address two challenges familiar to mathematics teacher educators. The first was limitations in teacher knowledge and practice. Research had shown that many middle school mathematics teachers have gaps in essential conceptual understandings necessary to facilitate CMA (e.g., Shechtman, Roschelle, Haertel, & Knudsen, 2010). Also, CMA requires social and sociomathematical norms beyond prevalent traditional teaching practices organized around teacher-centered interactions, such as Initiate-Respond-Evaluate (IRE; Cazden, 2001; Hiebert, Gilmore, & Stigler, 2002). The second challenge was the limitations of professional development (PD) approaches. Little research was available to indicate what types of in-service PD experiences would affect practice. Content knowledge was typically a focus of much PD, but researchers broadly agreed that this was necessary but not sufficient (Ball & Cohen, 1999; Ball, Lubienski, & Mewborn, 2001; Empson & Junk, 2004; Hiebert, Gallimore, & Stigler, 2002; Ma, 1999; Sowder, Philipp, Armstrong, & Schappelle, 1998).

We decided to focus on enabling middle school mathematics teachers to begin to make a transition from more traditional practices to setting up their classrooms to provide students with basic opportunities to engage in CMA. To do so, the model would need coverage of mathematical content integrated with practical and readily implementable teaching practices. Foundations of our workshop design were rooted in best core practices in teacher PD design (Loucks-Horsely, Love, Stiles, Mundry, & Hewson, 2003; Porter, Garet, Desimone, & Birman, 2003) and the growing literature on mathematical knowledge for teaching (MKT; e.g., Ball & Bass, 2000; Ball, Lubiensky, & Mewborn, 2001; Hill, Rowan, & Ball, 2005).

In addition, we sought to develop an approach that would build stable "bridges" from PD workshop to classroom practice so that teachers could take what they learned and readily apply it in their practice. We derived our "successive approximations of practice" approach from research in the "pedagogies of practice" literature, in which researchers characterize the composition and affordances of pedagogies for professional learning (e.g., Boerst, Sleep, Ball, & Bass, 2011; Grossman et al., 2009; Kucan et al., 2011; Lampert et al., 2013). Grossman et al. (2009) developed a framework of professional pedagogies that include three broad concepts.

1. *Decompositions of practice* make explicit the building blocks of practice by "breaking down of complex practice into its constituent parts for the purposes of teaching and learning" (Grossman et al., 2009, p. 8). In Bridging PD, we make explicit the building blocks of facilitating argumentation, including the mathematical structure of arguments and key facilitative teaching moves for CMA.
2. *Representations of practice*, like mathematical representations, are simulated ways of conveying abstract qualities of classroom practice. In Bridging PD, one type of representation used is scripts of classroom dialogue that teachers read aloud.
3. *Approximations of practice* use the representations in embodied workshop activities to allow novices to try out aspects of practice within a controlled situation in which mistakes and failure are expected. In Bridging PD, we provide approximations using several representations (e.g., scripts, role-plays).

Our approach also draws on research that conceptualizes teaching as "disciplined improvisation" (e.g., Borko & Livingston, 1989; Burnard, 2011; DeZutter, 2011; Sawyer, 2004, 2011). Researchers in this milieu connect the professional practices of teaching with methods used in improvisational arts such as music and theater, in which experienced performers produce their work creatively as an ensemble in the moment without a script (Johnstone, 1979;

King, 2001; Sawyer, 2011). They recognize the power of improvisational approaches to enable teachers to develop classroom learning environments that support productive collaboration and intellectual risk-taking (Lobman, 2007; Lobman & Lundquist, 2007), essential to making "bold conjectures" (Shechtman & Knudsen, 2011). Discipline is a critical component. Improvisation does not mean anything goes; it requires training, practice, and expertise in the underlying structures of the art form (Sawyer, 2011). Improvisers must be disciplined in their mastery of a shared toolkit that can be used creatively.

We will describe the design of the Bridging PD model and its implementation within the context of a 2-year small scale randomized impact study. Our driving research questions were: *What are impacts of Bridging PD on teachers' knowledge and the discourse in their classrooms? How do teachers use Bridging practices to facilitate argumentation?* We conclude with a discussion of implications for the mathematics teacher educator community.

The Bridging PD Model

This section describes each of the components of the 2-week Bridging PD model (Figure 1), starting on the right-hand side with the intended outcomes of the program.

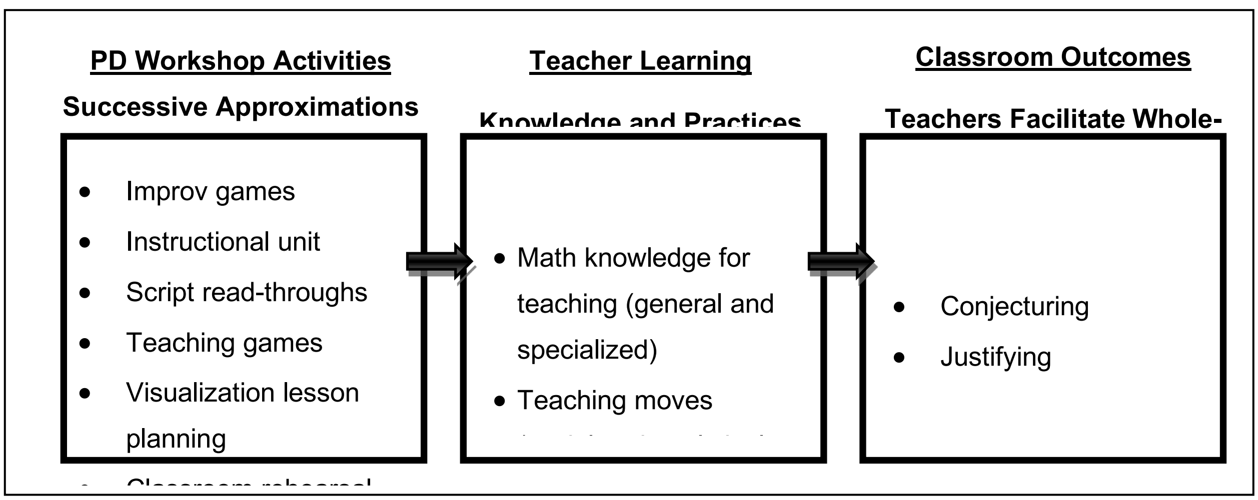

Figure. 1 Bridging PD Model

Classroom Outcomes: Teachers Facilitate CMA Discussions

Building on prior research (e.g., Erduran et al., 2004; Zohar & Nemet, 2002), we decomposed mathematical argumentation into a set of three explicit practices that could be addressed in classroom discussion. As shown in Appendix A, these practices are (1) *conjecturing*, a process of "conscious guessing" (Lakatos, 1976) to create mathematical statements of as-of-yet undetermined mathematical validity (or claims); (2) *justifying*, a process of explicating one's reasoning to establish the mathematical validity of a conjecture (thus combining Toulmin's (2003) data and warrant); and (3) *concluding*, the process of coming to agreement about the validity of the conjecture and its justification.

Teacher Learning: Knowledge and Practice to Facilitate CMA

Bridging PD addresses two broad categories of teacher learning. The first is knowledge of the mathematics, for which we use the construct "mathematical knowledge for teaching" (MKT; (e.g., Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Hill, Rowan, & Ball, 2005), including both general and specialized components. *General MKT for argumentation* comprises in-depth understanding of the mathematics students learn, such as the practices of argumentation and the mathematics content that is the subject of argumentation (e.g., representations, derivations of concepts, connections to related content). Specialized MKT for argumentation encompasses knowledge that would be particular to the work of teaching, such as how to evaluate the validity of students' mathematical conjectures and justifications, differentiate between colloquial and mathematical uses of language, evaluate students' statements for their mathematical precision, and interpret unconventional constructions (Shechtman et al., 2010).

For the second category, the practices necessary to facilitate CMA, our decomposition focuses on teaching moves. A "move" is conceptualized as a unit of instructional practice and is the "smallest chunk of behavior that can be aimed at a purpose" (see Jacobs & Ambrose, 2008; Mumme & Carroll, 2007). That moves are specific and behavioral helps teachers know what they can do. CMA teaching moves help students attend to and move through conjecturing, justifying, and concluding. They include both "social scaffolding," such as establishing and maintaining appropriate social norms and behaviors around discussion, and "analytical scaffolding," moves that orchestrate and facilitate mathematical argumentation practices (e.g., Williams & Baxter, 1996). For example, teaching moves may help students understand rules of participation in classroom discussion, make sure many students' conjectures are voiced, elicit examples or counterexamples, or conclude an argument. Moves may also include questioning to probe for knowledge or draw out a student's articulation of a conjecture or justification, or providing mathematical content that revoices a students' ideas or contributes to articulating a conjecture or justification. These moves are consistent with those emphasized in research on discourse-oriented mathematics teaching (e.g., Boaler, 2002; Boaler, 2008; Forman, Larreamendy-Joerns, & Brown, 1998; Lobato, Clarke, & Ellis, 2005; Pierson, 2008; Silver & Stein, 1996; Stein, 2001; Williams & Baxter, 1996; Wood, 1999; Yackel, 2002).

PD Workshop Activities: Successive Approximations of Practice

Bridging PD pedagogy comprises a strategically sequenced set of successive approximations of practice that enables teachers to experience, analyze, enact, and reflect on CMA and the practices that support it. For the teacher professional developers, the range of approximations enable facilitators to model, highlight, and focus systematically on key facets of practice, providing structured opportunities for group reflection and coaching. Approximations are activities that start with a focus on MKT and then integrate with increasing complexity opportunities to build facility in understanding and enacting classroom teaching moves. Activities culminate in a role-play of the classroom that reconstitutes practice as a whole.

Approximation 1. Early in the sequence, drawing on methods from improvisational theater (e.g., Lobman & Lundqust, 2007), we introduce traditional improv games to create a learning environment in which teachers can experiment with new practices in a safe space where mistakes are acceptable and even celebrated. One such game is Giving a Gift, in which players present each other with pantomimed gifts and tell stories about them. While this can seem somewhat intimidating to some teachers at first, most grow to enjoy them and appreciate the creativity and team spirit that they foster quickly.

Approximation 2. The next approximation of practice is the use of the instructional curriculum unit. A 4-day curriculum unit serves as an anchor for all subsequent activities and discussion. It provides a content foundation for argumentation, as well as seeds for CMA—opportunities to recognize patterns, generate conjectures, justify conjectures, investigate conjectures for their mathematical validity, and use counterexamples. As a distal approximation of practice, teachers engage with these student materials as adult learners as the workshop leaders model facilitation. This provides teachers the opportunity to deepen their general and specialized MKT, get a first-hand but low-risk experience of what it would be like to participate in CMA, and begin to develop confidence in CMA as a learning tool. Embedded in the workshop materials are “mini-cases,” scenarios in which teachers can begin to consider how students themselves might respond to some of the prompts in the activities. Teachers also receive a teacher guide that includes prompts they may use in facilitating discussion.

Approximation 3. The next approximation is reading through classroom discourse written as scripts. We prepared a set of scripts of classroom discourse around the curriculum materials, tailored to highlight particular aspects of specialized MKT and specific teaching moves. As an approximation moving closer to actual classroom practice, teachers read through the scripts, taking on roles of teacher and students. This provides teachers with accessible enactments of teaching in which practice is decomposed and scaffolded. The focus is on only the content of what teachers and students say, removing responsibility for the in-the-moment decision making that comes with teaching.

Approximation 4. The next approximation of practice is “teaching games.” Modeled after improv games, these are designed to provide teachers opportunities to begin to enact and reflect on specific teaching moves. Appendix B shows two examples. One is Closed-Ended Only and Open-Ended Only, in which teachers explore the actions and affordances of different types of questioning. One player takes the role of “teacher” and facilitates a “student” in justifying a conjecture, while asking only closed- (open-) ended questions. In teaching games, teachers take on a similarly sized chunk of teaching as in the script read-throughs, but the element of improvisationally responding to “students” makes the activity a closer approximation to practice. Teachers make decisions, not just say the words of others, yet the type of response they can make is still purposefully constrained. The rules of the game lend focus to an improvisational activity, affording new opportunities for learning what each type of question is good for.

Approximation 5. The next approximation is visualization lesson planning. Expert versus novice teacher studies indicate that expert teachers tended to make extensive use of mental plans instead of relying on written plans because they had well-organized knowledge of pedagogy (Borko & Livingson, 1989; Clark & Yinger, 1979). In Bridging PD, teachers create written plans collaboratively. They are asked to visualize a lesson and what they would be doing and saying, and what students would likely do and say as a result. They “think aloud” as they visualize, approximating the practice of expert teachers, and a partner teacher takes notes. The teachers then turn these visualizations into detailed plans that structure the next activity (rehearsal). This sort of planning is based on imaginings that may be fairly near approximations to practice but more elaborated than what would typically occur in actual practice.

Approximation 6. The culminating approximation of practice is a classroom rehearsal. As in an improvisational performance on stage, teachers facilitate an episode of conjecturing or justifying in a simulated classroom environment. Teachers take turns being the "teacher" and having the rest of the group be the "students." This provides a dynamic recomposition of teaching moves and MKT into one experience that closely approximates classroom teaching. Teachers can also debrief extensively, coaching one another with support of the facilitator, exploring together what works and does not work in a dynamic classroom setting.

Research Methods

The driving research questions were: What are impacts of Bridging PD on teachers' knowledge and the discourse in their classrooms? How do teachers use Bridging practices to facilitate argumentation? Bridging PD was implemented in the context of a 2-year small-scale randomized experiment impact study. The main outcome measures were teacher MKT and the CMA in the teachers' classrooms. We also conducted a qualitative investigation of how teachers used teaching moves in their classrooms to facilitate argumentation.

Implementation of the Bridging PD Program

The Bridging PD program was implemented as a 2-week workshop each summer. Week 1 (24 hours) focused on development of MKT through engagement with the second approximation of practice, the 4-day instructional curriculum unit. Week 2 (30 hours) focused on facilitating CMA. Teachers engaged in the rest of the sequence of approximations to build "bridges" from the workshop to the classroom. The agendas can be found in Appendix C. Across the two years of the project, we focused on two different middle school content areas. Sample lessons can be found in Appendix D. Note that there was also iteration across years in the workshop design. In Year 1, the content was proportionality, connecting number and operations, geometry, and algebra, (Hiebert & Behr, 1988). In Year 2, the content was coordinate geometry, including the history, rationale and conventions for the coordinate plane, definitions of lines and rectangles, and finding patterns among coordinates of geometric shapes.

Experimental Design

Teachers were randomly assigned to a Treatment or Control group. Assignment was at the school level so that teachers could work together in community. Treatment teachers received the full 2-week Bridging PD. Control teachers participated with the Treatment teachers during Week 1, which could be attended as a stand-alone workshop. In Week 2, Control teachers were provided a different PD experience—vertical coordination of mathematics standards, assessments, and curricula across grade levels (e.g., College Board, 2006; College Entrance Examination Board, 1998). The Control training was intended to provide professional benefit comparable to that of the Treatment training, while serving as a counterfactual that did not address CMA or use approximations of practice. All teachers in both groups were asked to teach the 4-day Bridging PD unit during the school year at a time that fit appropriately within their scope and sequence. There were two main hypotheses:

1. Teachers in both the Treatment and Control groups will show growth in MKT from pre- to post-workshop.
2. In classroom discourse observed during the Bridging PD units, there will be more CMA talk in the classrooms of Treatment teachers than in the classrooms of Control teachers.

Outcome Measures

MKT assessments. We developed one assessment for each year. Each took 60–90 minutes and assessed the MKT necessary to teach the instructional unit. They contained a mix of multiple choice questions and items requiring the construction of mathematical definitions and justifications. The development process was as follows: (1) develop a conceptual framework of content covered; (2) hold a 1.5-day “item camp” with the Bridging PD team to create a pool of potential assessment items; (3) conduct cognitive think-alouds of the items with 3 teachers known to represent a range of prior MKT; (4) field test the assessment with about 15 teachers to determine item characteristics (e.g., percent correct, ceiling effects, response variations) and test length. The Year 1 assessment (proportionality) had 23 items and a Cronbach’s alpha of .84. The Year 2 assessment (coordinate geometry) had 30 items and a Cronbach’s alpha of .94.

CMA talk and teaching moves during whole-class discussion in the Bridging PD unit. We conducted classroom observations during two key unit lessons (two days in most cases). One or two observers videotaped the lessons, using high fidelity microphones to capture teacher and student talk during whole-class discussion. We developed a coding protocol to characterize teachers' CMA. The protocol is applied to verbatim transcription of whole-class discussion. Each year, a team of coders (three in Year 1 and four in Year 2) coded the transcripts; each transcript was coded by two coders.

The protocol requires two passes through a transcript. In the first, coders identify all of the episodes of CMA talk. Coders locate an episode by identifying an "entry," "body," and "exit" of a block of discourse focused on argumentation. An episode entry was most typically a teacher's prompt for a justification. The episode body would thus be the process of justifying a conjecture and the exit would be whatever utterance(s) demarcate the ending of this event and shift to the next one. In the protocol, two coders independently determine the utterances at which they observe entry and exit points for CMA episodes. The coders sought consensus through discussion. This process does not afford a metric of inter-rater reliability.

In the second coding pass, within each episode of CMA, coders use codes to characterize each turn of student and teacher talk, yielding counts of the number of student statements and the number of key types of teaching moves. We chose to focus on teaching moves that were both essential to facilitating CMA practices and objectively codable. Table 2 shows the codes and their inter-rater reliabilities. In Year 2, based on our Year 1 findings and new research that had recently been completed (see Pierson 2008), we enhanced the coding scheme by examining teacher "demand" and "give" moves of differing intellectual work. Note that because we refined the coding process between years, we used slightly different coding processes and reliability metrics each year (see Table 1 notes).

Table 1. Codes for each turn of CMA talk.

Code	Description	Inter-Rater Reliability
Student statement	Student takes a turn of mathematical talk.	0.98 (Y1)[a] 95.3% (Y2)[b]
Teaching moves Year 1		
Orchestration move	Teacher solicits participation, consensus, agreement, examples, or counterexamples.	0.85 (Y1)[a]

Closed-ended questions or prompts for closed-ended responses	Teacher question/prompt that can be answered with a yes/no response or a specific piece of information (e.g., calculation, recall of fact or procedure).	0.96 [a]
Open-ended questions or prompts for open-ended responses	Teacher question/prompt that cannot be answered in a closed-ended manner (e.g., “Why,” “Explain,” “How do you know?”).	0.75 [a]
Teaching moves Year 2		
Orchestration move	See Year 1	93.4% (Y2) [b]
Questions demanding low intellectual work	Teacher request to recall facts, confirm agreement, read values off representations, perform an arithmetic calculation, perform an automatic procedure, repeat previous comments.	87.6% that a teaching move was demand [b]
Questions demanding high intellectual work	Teacher request to explore and extend ideas, explain, justify, compare, contrast, interpret, provide examples and counterexamples, make connections, generalize.	80.2% that, given it was demand, it was low or high [b]
Give information with low intellectual work	Teacher provides information that is the result of recall, reading values off of a representation, performing an arithmetic calculation, evaluating answers.	81.6% that a teaching move was a give [b]
Give information with high intellectual work	Teacher provides conceptual information that serves to extend, challenge, or explain a big idea (e.g., explaining, justifying, providing counterexamples, interpreting, predicting).	89.4% that, given it was a give move, it was low or high [b]

[a] In Year 1, inter-rater reliability was determined by calculating the intraclass correlations among the coders' counts of the number of turns of talk they determined to be coded with a particular code.

[b] In Year 2, inter-rater reliability was determined by calculating the percent of turns of talk, line by line, that coders agreed should be coded with a particular code.

Qualitative Analyses of Classroom Discussions

We also investigated how teachers configure teaching moves holistically in their practice of supporting CMA. Building on what we learned from the teaching moves codes, we qualitatively examined classroom transcripts, examining the flow of CMA in various discussions. We focused on how teachers used teaching moves in different ways to position themselves in the classroom, give students mathematical authority, and scaffold student mathematical thinking.

Recruitment and Participants

We recruited middle school mathematics teachers from urban districts in the San Francisco Bay Area. We believed it was important to do our work in high-needs areas, even though factors faced by teachers there often lead to teacher attrition (Ingersoll, 2001). Strategies included sending recruitment fliers to all eligible teachers, short presentations at faculty meetings, and follow-up phone calls. All teachers who applied were accepted. One teacher from a private school also joined in Year 2. In Year 2, 9 of the 21 teachers had participated in the Year 1 study, and 5 of these teachers completed the Year 2 research. Teachers who attended for the second year remained in the same experimental group. Table 2 shows the demographic information for teachers who completed all of the research requirements.

Attrition was high in Year 1 and lower in Year 2. In Year 1, a total of 25 teachers attended the workshop, and 15 of them completed the research requirements (40% attrition). While attrition was not different across experimental groups, teachers who completed the study compared to those who did not had lower MKT ($p < .05$) and taught in schools that on average had a higher percentage of students eligible for the lunch program ($p < .01$). In Year 2, a total of 21 teachers attended the workshop, and 16 of them completed the research (24% attrition).

Table 2. Demographic information for participants who completed all research requirements.

Variable	Year 1 (2006)	Year 2 (2007)
Teachers		
Count		
Total	15	16
Treatment, Control	9,6	8,8
Female (%)	73	69
Years teaching total		
Mean	8.1	5.8
Range	1.5 – 20	1 – 16
Teacher ethnicity (%)		
White	53	38
Hispanic	0	15
Asian	27	31
African American	20	15
Master's degree (%)	36	25
Schools		
Total count	9	8
Eligible for lunch program (%)*		
Mean	44	50
Range	0 – 76	68 – 83

Student ethnicity (%)*		
White	18	9
Hispanic	18	34
Asian/Pacific Islander	20	15
African American	41	37
English Language Learners (%)*	10	22

* No significant differences were found between Treatment and Control groups on any of these variables within each year.

Note: School data does not include the private school in which one Year 2 teacher worked.

Results

Experimental Impact Study

Hypothesis 1: Increased MKT. This was supported in both years (Figure 2). Teachers came to the workshop with a wide range of baseline MKT. From pre- to postworkshop, teachers' mathematics learning on average was significant, though modest; it grew a mean of 0.5 *SD* [$t(23) = 4.2$, $p < .001$] and 0.3 *SD* [$t(20) = 4.5$, $p < .0001$] in Years 1 and 2, respectively.

Year 1

Proportionality

Year 2

Coordinate Geometry

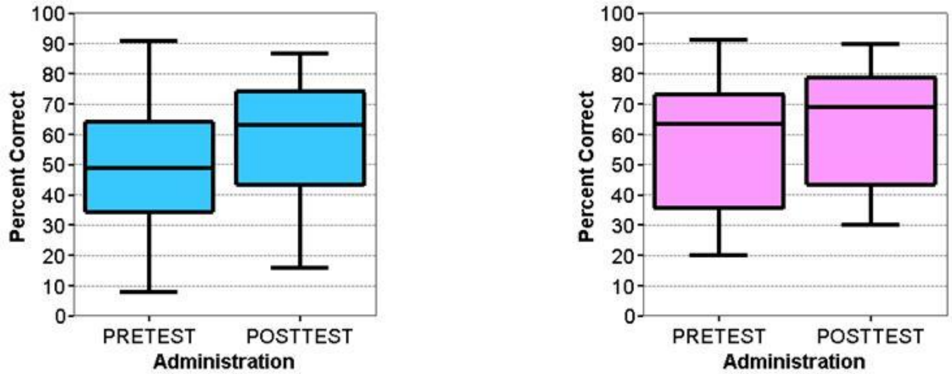

Figure 2. Box plots of distributions of teacher MKT each year.

Hypothesis 2: Increased CMA. This was supported in both years—with statistical significance in Year 1 and marginal significance in Year 2. Note that findings across years are not directly comparable; differences in curriculum materials provided different types of supports for argumentation, and the coding schemes were different across years. Specific findings were:

- *Episodes of argumentation.* In Year 1, coders found a mean of 14.7 (*SD* = 4.2) episodes in Treatment classrooms and 9.7 (*SD* = 3.9) in Control classrooms. The difference between groups was statistically significant [$t(13) = 2.3$, $p < .05$]. In Year 2, they found a mean of 3.2 (*SD* = 1.9) episodes in Treatment classrooms and 3.0 (*SD* = 2.8) episodes in Control classrooms. That difference was not statistically significant [$t(14) = .21$, $p = .84$, n.s.].
- *Student CMA talk.* Figure 3 shows that in both years, while some student CMA talk occurred in Control classrooms, more occurred in Treatment classrooms. In Year 1, this difference was statistically significant [$t(13) = 2.6$, $p < .05$], while in Year 2, this difference was a marginally significant trend [$t(14) = 1.4$, $p = .18$]. Note that it we expected some CMA talk in the Control classrooms, as teachers used the same curriculum materials that seeded argumentation and were exposed to the discourse in their MKT workshop.

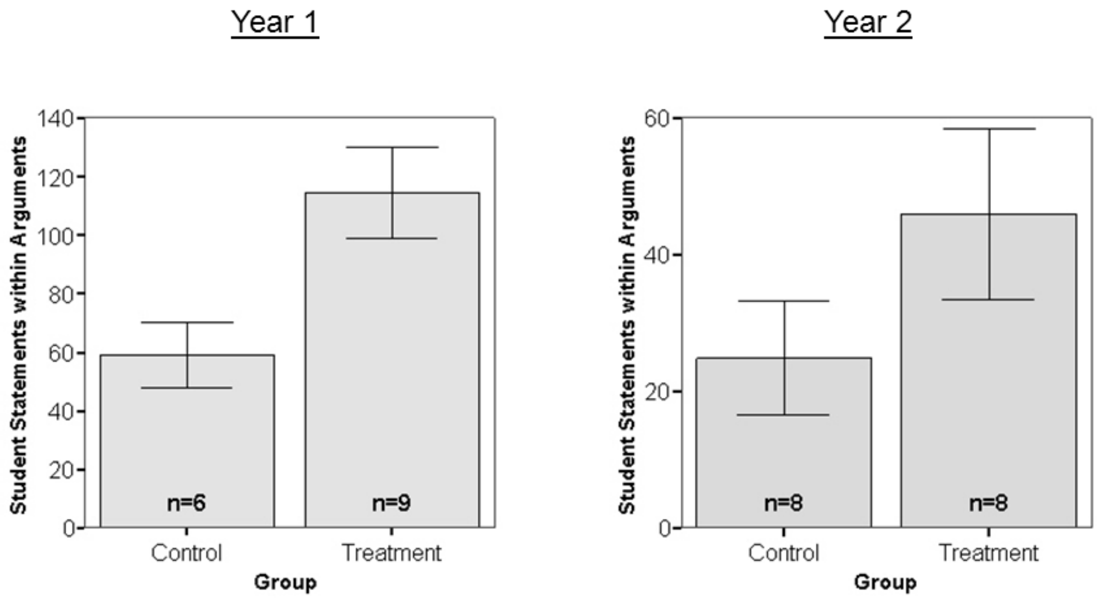

Figure 3. CMA talk in the classroom: mean counts (± standard error) of student statements in whole-class argumentation. Note: Group differences were significant in Year 1 and marginally significant in Year 2. Because curriculum materials were different, cross-year comparisons are not meaningful.

- *Teaching moves.* Figure 4 shows the counts of teaching moves during the episodes of argumentation. In Year 1, Treatment teachers did more of all the teaching moves that were coded: closed-ended questions [$t(11.5) = 2.2$, $p < .05$], open-ended questions [$t(12.0) = 2.8$, $p < .05$], and orchestration moves [$t(11.8) = 2.4$, $p < .05$] (each of these *t*-statistics is adjusted to accommodate unequal variances between groups). In Year 2, there were nonsignificant trends that Treatment teachers did more of each of the examined teaching moves. This finding highlights teachers' engagement and support in the CMA discourse with their students and that they used a variety of teaching moves to facilitate these classroom discussions.

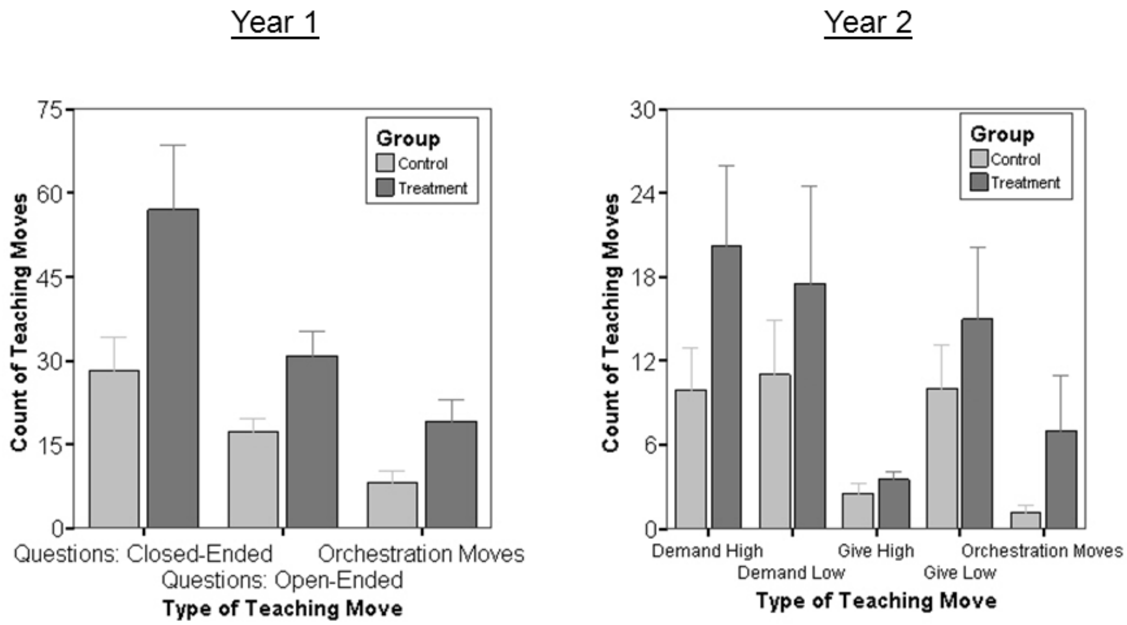

Figure 4. CMA talk in the classroom: mean counts of analytical scaffolding teaching moves (± standard error) in whole-class argumentation. Note: Group differences were significant in Year 1 but not in Year 2. Because curriculum materials were different, cross-year comparisons are not meaningful.

Another important finding was that in the Treatment group, teachers' MKT was a significant predictor of the amount of argumentative talk in their classrooms. MKT was correlated with the number of student statements in Year 1 [$r(7) = .80$, $p < .05$] and in Year 2 [$r(9) = .91$, $p < .01$]. In the Control groups each year, these correlations were small and non-significant. This finding suggests that PD on teaching moves enabled teachers to better use their mathematical knowledge in their teaching to support CMA.

Teaching Moves in Practice: The Orchestrator and the Socratic Questioner

For this report, we present analyses from the classrooms of two teachers who illustrate contrasting ways of using teaching moves to facilitate CMA. Teachers Stephanie and Esther both had substantial argumentative talk in their classrooms, and they both used a rich blend of the teaching moves emphasized in the Bridging PD. For each teacher, we developed these examples by examining their transcripts. We first looked at how they were coded using the protocol. We then examined the transcripts in greater depth to characterize narratively the flow of teaching moves and student argumentative talk as the classes moved through conjecturing, justifying, and concluding. Note that we do not have data to determine whether these teachers learned these teaching moves in the Bridging PD; these are simply illustrations of how teaching moves can be used.

Teacher Stephanie: The Orchestrator. In general, this teacher's approach was to place herself into a role we characterize as one of *orchestration*, allowing students primarily to drive the discussion with her guidance and direction.

Overview. Stephanie participated in Year 2 of the program (she did not participate in Year 1), using the unit on coordinate geometry with her sixth grade advanced math class. Importantly, this was only her second year of teaching experience, and she had adopted a set of classroom management skills that she used with some success. In terms of CMA, a snapshot of her classroom is: (1) compared to many other treatment teachers, she had extensive student argumentative talk; and (2) her teaching moves were primarily orchestrations of participation and requests for high intellectual work.

Mathematical activity. In this activity, entitled "Making Rectangles II" (see Appendix D), students draw a coordinate plane and label the axes. They then draw four rectangles in different places, covering all four quadrants, labelling the vertices. They then make arguments about the patterns in the vertices. In Stephanie's class, we observed this lesson spread over 2 days of instruction. On the first day, Stephanie began with a review of the conventions of a coordinate plane and then had students work individually to draw and label their axes and rectangles. Students then worked in pairs to look for patterns in the coordinates.

Conjecturing. Stephanie had the pairs come together into groups of four to use the patterns they found to make conjectures about coordinates. Each group wrote their conjecture on a poster that they then presented to the class. She then had the class vote from a choice of four conjectures which one they wanted to justify as a class.

It is at this point that the whole-class discourse moved into improvisational CMA. The class first voted for the conjecture, "If there are four rectangles on the grid, then they must face the same way." She then launched the class into a subphase of conjecturing, clarifying, in which the class had a discussion for the rest of the period to come to a shared understanding of what this meant. The next class period, to start the discussion freshly, she had the class vote for another conjecture. This time the winner was, “If it’s left and below the origin, then all the numbers are negative.”

We explored the discourse in the 23-minute CMA session that followed. The students were apparently quite eager to do argumentation, asking for it and cheering when Stephanie announced they would do it. As they transitioned into CMA, Stephanie placed herself in primarily a facilitator role, outside of the argument, orchestrating participation. Many of her teaching moves served to help students understand where they were in the CMA process, moderate how students participated with one another, and keep the conversation going. For example, at the beginning:

STEPHANIE: We are *clarifying* what that means. So you understand what we are doing. We are clarifying right now. Um, Andy.

ANDY: So pretty much that if it’s like, if it’s in the negative y-axis, then the numbers have to be negative. Right? That’s what it’s saying?

STEPHANIE: *(To Jason)* He’s asking *you*, because I’m not in this. Jason.

JASON: Not really. They are saying like if, if you have a coordinate grid, and the point is to the left and below of the center point, the origin, then both numbers have to be negative.

After several more rounds of clarification among students, some based on a coordinate grid displayed on the board, Stephanie moved to end clarification:

STEPHANIE: Ok, I’m going to offer a suggestion. If you think you have a clarification of that statement, you can write it more clearly, go ahead and write it and then present that, so we can move, we can say, yeah that’s clarified and move on.

Taking up that suggestion, a student made a diagram on the board to help him summarize what others had said. The diagram showed the coordinate plane with PP, NP, NN, and PN written in quadrants I through IV, respectively, to indicate which coordinates would be negative and positive. He narrated as he drew, setting the context of conjecture in the coordinate plane. This type of discussion continued for several minutes, as students wordsmithed the conjecture and checked each other's understandings.

Justifying. Stephanie's next moves were to end the clarifying stage and begin justification. Note that another move was giving students "points" for math argumentation, encouraging participation from all students.

STEPHANIE: Put your hand up if you don't understand what the conjecture is. *(No hands.)* Okay let's move on then to justification. Now you are going to argue that this is true or this is not true. *(Several students raise hands.)* You are going to argue this is true or not true. Remember to get your points for math argumentation, you need to be participating. Vanessa.

VANESSA: I think it's true because they said that this is, I think it's true because of the positive up there on the right side, the negative number is left side.

STEPHANIE: Okay, so you think it's true? *(Several students raise hands.)* Harrison.

HARRISON: Um, I think it's also true because you have your coordinate plane, and this is the negative, negative, corner right? *(points to quadrant III)* To make, uh, to make it into a positive, then you'd have to bring up here. *(points to quadrant II)*

Vanessa was providing justification that coincided with part of Jason's clarification; she addressed the location of positive and negative numbers on one axes. Harrison pointed out that the points in the lower left had to be negative since the only way to get a positive number would be to shift to a different quadrant. Vanessa and Harrison's statements, while related, were not explicitly connected. There were several more partial justifications, based on Jason's diagram, until a student added an "unless" clause to the argument:

JESSE: …so unless you have some wacky coordinate plane, like positive over here, wacky and not real, then it's true.

Jesse introduced the idea of a nonconventional coordinate system in which the conjecture would not be true. He seemed to be using it as further justification of the conjecture. Another student added:

KENNYN: I think it's true, and I agree with what Jesse said. If it's a wacky coordinate plane it wouldn't. Since that *is not* what we are using, I think it [the conjecture] is true.

Stephanie added an "unless" clause to the conjecture on the board. Other students argued for the existence of an alternative labelling for the coordinate system. They said that the axes on the board could be rotated and so the positions of positives and negatives would change. Others contested these ideas and said either that there was no other way to label the plane or that they were limited to working with the system illustrated on the board and in their math books.

Concluding. Eventually, Jason made a bid to conclude. Stephanie interjected a request for a counterexample:

STEPHANIE: Ok. [Jason] says everyone thinks it's true. Does anybody have, does anyone think it's not true? Does anyone have an example of where it is NOT true? Tina?

Several more students posed alternatives to the standard coordinate system of labels, and other students argued against them. Interest appeared high, but the arguments began to be rehashed. Then Stephanie intervened with a process for concluding. Rather than ask students explicitly for agreement, Stephanie's move was to look for lack of disagreement. After one student threatened another, Stephanie chose to conclude by class vote: who agreed and who disagreed with the revised conjecture. Even then, students continued to chatter about the coordinate grid and how it was labelled.

Analysis. Throughout this episode of CMA, we can see how the coded snapshot played out; as a facilitator external to the argument, Stephanie orchestrated participation and asked high-demand questions. Her orchestration moves were typically simple, straightforward explanations or reminders, and they served to launch students into making fairly lengthy mathematical statements. She reminded students of behavioural expectations, "passed the baton" among students, and explicitly took herself out of the discourse. She also occasionally asked questions, such as calling for a counterexample, encouraging students toward more sophisticated mathematical thinking.

The class was engaged in CMA in accordance with our model. The students proceeded through the stages, each with meaningful discussion: they made or contributed to arguments in multiple sentences, and they engaged and responded to each other directly. Ending the argument through a vote is not a mathematical move, though it was one that was mentioned in the workshop. Still, in balance, this was a robust example of CMA from a teacher who was very likely new to the process.

Teacher Esther: The Socratic Questioner. This teacher, in contrast, positioned herself in the center of the argumentation content itself, facilitating students' reasoning processes.

Overview. Esther was a teacher in Year 1 of the program, using the unit on proportionality with her class that mixed sixth through eighth graders. She was a veteran teacher with many years of experience who discussed in the workshop that she already had a fair amount of experience with "discovery"-type methods, though the framework of argumentation was new to her. A snapshot of the CMA in her classroom is: (1) compared to many other treatment teachers, she had extensive student argumentative talk; and (2) her teaching moves included notably extensive questioning, both closed-ended and open-ended.

Mathematical activity. In this activity, "Making Similar Rectangles" (see Appendix D), students created a set of five paper rectangles, all similar to each other. They were encouraged to use and describe more than one method for making them. The materials explicitly stated the conjecture, “All the rectangles in this set are similar to each other,” and instructed students to justify this conjecture in two different ways. Esther conducted this activity over 2 days.

Conjecturing. On the first day, students worked in small groups, each around a table. They worked with a variety of materials (e.g., paper, scissors, tape) and methods (e.g., folding, measuring) to create similar rectangles. Conjecturing was not made explicit in this activity; rather, the methods used to make the rectangles were meant to inform the students’ understanding of the common conjecture.

Justifying. On the second day of the activity, Esther called on one group at a time, talking with a “spokesperson” to give that group’s justification of the common conjecture. Rather than one large whole-group discussion, as in Stephanie’s case, this led to several small episodes of justification. Here we look closely at one example. The argument began:

SAM: I don’t know how we came up with this exactly, but I guess we figured that it’d be the easiest rectangles to make similar. So 1 by 2, to 2 by 4, to 3 by 6, to 4 by 8. And if you noticed all of these they are just getting bigger by 1 each time.

Esther followed up with a series of questions that afforded the opportunity for the students to elaborate on their justification and clarify their mathematical thinking. She challenged the relevance of “getting bigger by 1.” She may have been concerned about students arguing from a common additive misconception, that adding the same amount to each side of a rectangle would result in a similar rectangle. She asked them to consider the other dimension. The student described what happened to the lengths in terms of adding, but not in terms of adding the same amount as to the width:

ESTHER: I don’t believe that. Oh, and what about the length?

SAM: The length is getting bigger by 2.

When the ratio of sides is 1:2, then adding 1 to one dimension and adding two to the other does result in similar rectangles. The student had thus made a mathematical justification for those particular rectangles being similar. But Esther challenged the generality of the group's argument:

ESTHER: Is that relevant for similarity?

SAM: Yes.

ESTHER: It is? You mean if I take any rectangle and increase the length by 1 and width by 2, I'm going to get similar rectangles?

SAM: No…no.

ESTER: So why are those similar?

SAM: Because…

ESTHER: Is it the inches?

One of the students found another way to state the relationship between 1 and 2 that was multiplicative, the final statement in the argument:

SAM: If you, if you uh, simplify any of these rectangles, they get to be 1 over 2.

ESTHER: 1 on top?

SAM: 1 by 2.

ESTHER: And?

The “And?” question was answered with another method they used to show similarity, but this particular justification ended with the question.

Analysis. Again, we can see how the coded snapshot played out; Esther’s teaching moves were dominated by extensive questioning with each group. She used questions one after the other to elicit clearer explanations or to address “holes” in a group’s argument. She questioned the generality of a group’s methods and pressed for further explanation of why their observations and methods justified similarity. She also orchestrated conversation by drawing in other students. Most of Esther’s questions were open-ended, but she also used closed-ended questions. Students’ replies ranged from one-word to several-sentences answers.

The class was engaged in CMA in accordance with our model, though the form of the arguments was quite different than in Stephanie’s class. Justification was drawn out by Esther’s questioning. Typically, one or two students spoke for each group. These students were involved in extensive argumentative discourse, and others occasionally joined in at Esther’s invitation. Conclusions were not elaborate, consisting mostly of a transition to other justifications, often with Esther’s expression of approval. Another robust example of CMA, argumentation happened with each group, students made complete mathematical statements that made sense in the context of the argument, and Esther clearly probed students’ thinking.

Discussion

Our experimental findings suggest that the Bridging PD approach shows promise to impact classroom practice. The PD led to increases in both teachers’ MKT and the CMA discourse in their classrooms. From pre- to postworkshop, we found modest shifts in teachers’ knowledge of proportionality in Year 1 and coordinate geometry in Year 2. We then found that the classrooms of teachers in our treatment group had more CMA talk, in terms of both student statements and teaching moves in support of CMA. Teachers used a variety of types of teaching moves to facilitate argumentation—open-ended and closed-ended questions, providing information at different levels of complexity, and orchestrating participation among students.

These findings suggest that mathematics teacher professional developers can use or adapt the successive approximation of practice approach to enable teachers to make basic shifts from traditional to more discourse-oriented practices. The model addresses teachers’ common and specialized content knowledge through the traditionally used vehicle of curriculum experiences;

it was particularly important that teachers experience making arguments themselves and with their peers. Building on this approach, Grossman et al.'s (2009) pedagogies of practice framework, typically applied to preservice teacher education, provided a useful framework for a deeper analysis of teacher practice and pedagogies to support its development. We were able to leverage the notions of decompositions, representations, and approximations of practice to characterize the essential practices to facilitate CMA and create a strategically planned set of activities to build these practices. Our approximations of practice had teachers engage in instructional units, scripts of discourse, teaching games, visualization lesson planning, and rehearsals of classroom discussion. Decompositions of practice were the essential repertoire of teaching moves teachers could use improvisationally to support argumentation.

Our data also inform expectations for how teachers may use teaching moves to facilitate CMA in their classrooms. We identified two kinds of approaches. The "Orchestrator" positioned herself primarily outside of the mathematical talk, affording her students to take on authority in a lengthy whole-class discussion. The "Socratic Questioner" provided more explicit teacher-centered practice drawing out student thinking. These two examples show that supporting CMA can be accomplished with different styles of teaching; our PD did not constrain these qualities. Also, the correlation between MKT and the number of teaching moves in the treatment group only provided further substantiation that MKT is *necessary* but not *sufficient* for teachers to enact more effective classroom practices. Indeed, professional development leaders must continue to attend to integrating these two types of knowledge and help teachers develop their own styles of positioning themselves as facilitators in the classroom.

Bridging PD also points to the potential of using improv theater methods for supporting teachers' improvisation in the classroom. We used improv games as both activities and inspiration for new teaching games. While we did not set up our inquiry to identify which elements of the Bridging PD model had what effects on teachers, our experiences with the teachers suggests that the "culture" of the improv environment that encourages productive failure can help teachers quickly get through some of their inhibitions around interacting with each other and eventually their students in new and uncertain ways. This is a growing area of research in general; since the Bridging PD program was implemented, applied improvisation has become increasingly popular in education, business, and personal development (e.g., Sawyer, 2011).

Conclusion

The adoption of new standards to prepare students to be career and college ready has resulted in urgency to teach core mathematical practices, creating a broad need for PD that enables teachers to understand and teach argumentation. More fundamentally, mathematical argumentation is an important epistemic practice for empowering students to construct their own understanding, develop intellectual autonomy and confidence in doing mathematics, and experience authentic mathematics. The Bridging PD's successive approximation of practice approach contributes to the repertoire of pedagogies that mathematics teacher educators can use to enable teachers to make basic shifts from traditional to more discourse-oriented practices. We have described in this article how it can address a fundamental challenge in teacher learning: developing both content knowledge and pedagogy for to support complex classroom mathematical discourse.

Peg, a veteran teacher in urban districts, after her first year of Bridging PD, summed up her experience in a way that spoke for many of our teachers: "If they can't make math arguments then they're doing algorithms, and they're going through [the] process. But if they can justify their thinking, then they own it. That's what I was trying to do this year."

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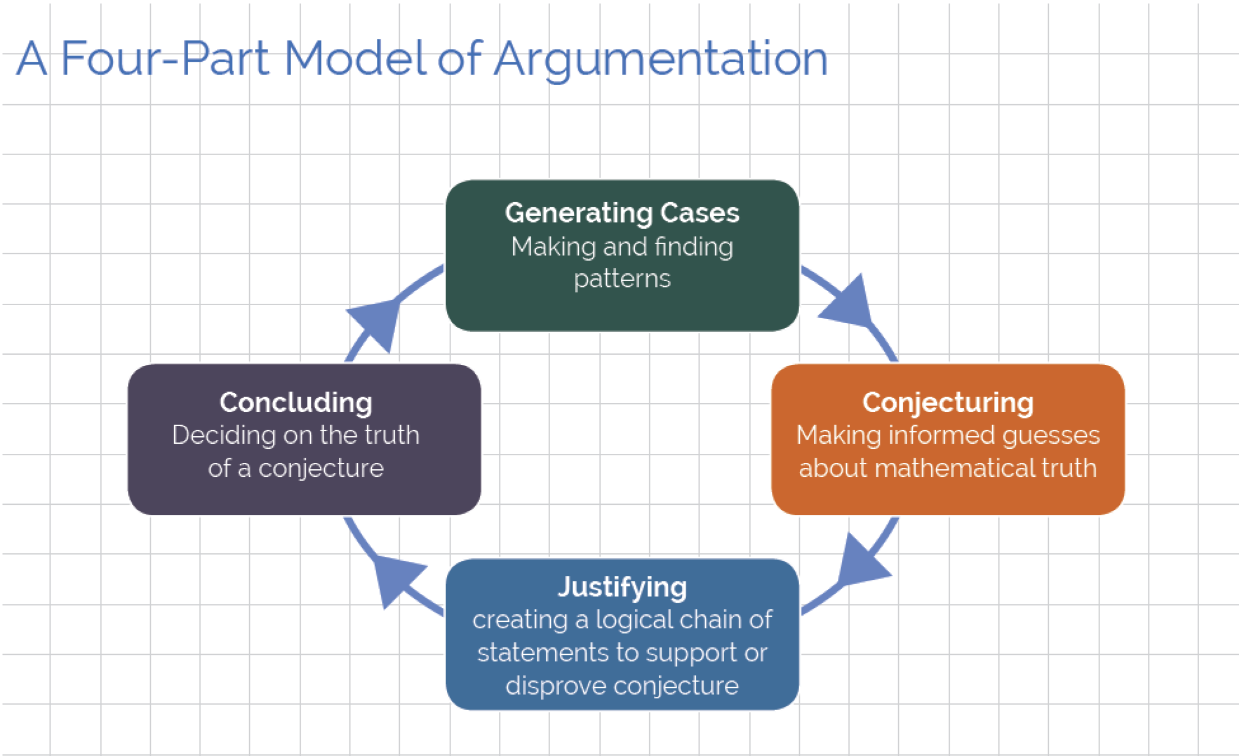
Appenedix A: Bridging PD Argumentation Framework

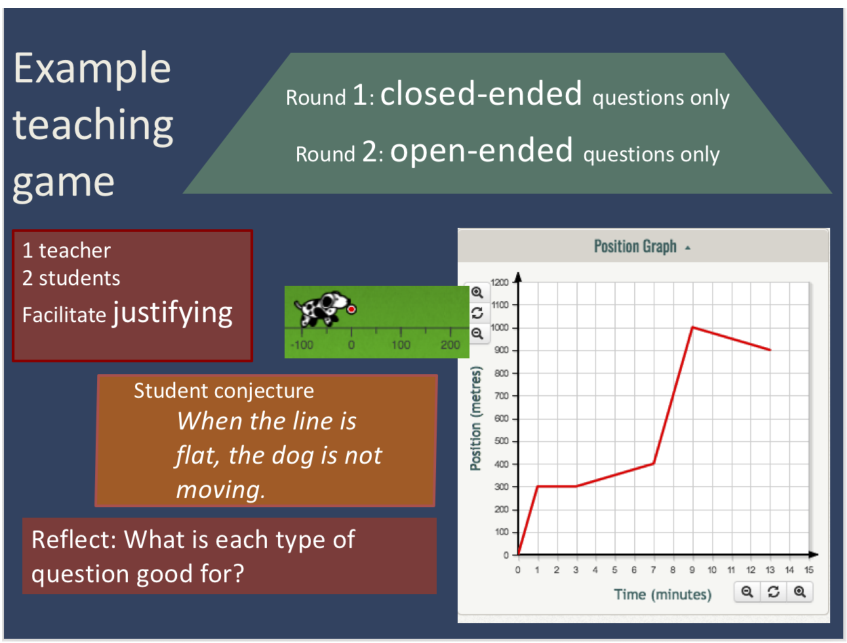


This model structures students' and teacher work together; it is used in both workshops and the classroom. Although there is a logic to the cycle and it typically starts with generating cases, argumentation does not always neatly follow the cycle. Teachers and students can focus on learning each stage separately. For example, if a teacher wants to focus students on learning how to justify, she may provide pre-made conjectures. If, on the other hand, she wants students to practice the art of making conjectures, she can provide cases for students to analyze, see patterns, and make informed guesses about what they think is a mathematical truth. Sometimes, generating cases happens before conjecturing, but sometimes students generate cases to make sense of someone else's conjectures. Finally, justified arguments often lead to additional conjectures.

Appendix B: Two Example Teaching Games

These two teaching games enable teachers to focus on practicing specific teaching moves. Teaching games have rules that define what the teacher should accomplish and the set of moves they can use. The games allow teachers to try out moves rather than merely discussing them. After each game, the facilitator leads a discussion about what moves worked for what purposes.

Closed-Ended Only and Open-Ended Only



In this game, teachers work in groups of 3-5. They choose a conjecture to justify. One “teacher” guides the “students” through a justification of the conjecture using *only open-ended questions*. The round is ended when everyone agrees on the truth or falsity of conjecture. Then teachers play another round with a new “teacher” and a new conjecture. This time, teacher guides students through a justification of the conjecture using *only close-ended questions.* Teachers then discuss the affordances and drawbacks of each type of question.

Why? Why? Why?

Roles: 2 arguers, 1 observer

Rules

1. Groups of three.
2. Choose two shapes to show similarity using cross multiplication
3. One person starts to describe the process of performing cross multiplication
4. The other asks Why at every opportunity.
5. Keep going until all are convinced of why “it works.”
6. Observer: what kind of justifications are given?

In this game, teachers explore the utility of the question “why” and how to judge when reasoning is sufficient. This page is from the Week 2 presentation. Teachers explored explanations of why shapes were similar or not similar.

Appendix C: Example Workshop Agendas

This Appendix provides examples of daily agendas for each of the workshops. Week 1 (24 hours) focused on the first approximation of practice, a 4-day instructional curriculum unit. Week 2 (24 hours) focused on the three remaining approximations of practice. The first agenda shows the types of activities teachers did on their first day of the Week 1 workshop, focused on teachers making their own mathematical arguments. The second agenda shows the types of activities teachers did on the second day of the Week 2 workshop, focused on pedagogy supporting argumentation.

DAY 1

"PLANE" OLD GEOMETRY

8:30 – 12:00	Research tasks and information
12:00 – 12:30	Lunch
12:30 – 1:00	A1. Mathematical argumentation in coordinate geometry
1:00 – 2:00	A2. Similar figures: How & why?
2:00 – 3:00	A3. Definition of similarity: What matters?
3:00 – 3:30	A4. Finding similar figures (Minicase 1)
3:30 – 3:35	Daily Evaluation

Note: Homework reading assigned today

Week 2: Day 2

8:30 - 9:00	Intro day 2
9:00 - 10:00	Teacher/student scenario “what comes next?”
10:15 - 12:00	Teaching games with teaching roles
12:00 - 1:00	Lunch and sun
1:00 - 1:30	Planning
1:30 - 1:45	Improv energy builder
2:00 - 3:00	Reality check + daily eval

Appendix D: Sample Lessons from the Bridging PD Curriculum Units

The following examples are from the two units. The first is from the Year 1 unit, *Similar Rectangles: Proportionality in Geometry, Number and Algebra*, and the second is from the Year 2 unit, *Rectangles in the Coordinate Plane*.

Making Similar Rectangles

1. Make a set of at least 5 rectangles, all similar to each other. Use more than one method for making them. Use different tools!

2. Describe your methods for creating your similar rectangles.

3. Now we have a conjecture:

All the rectangles in this set are similar to each other.

Justify this conjecture, two different ways.

Making Rectangles II

In this activity, use only rectangles that have sides parallel to the axes.

1. Draw an x- and a y-axis and label the units. Do not put the origin in the middle of the page!

2. Draw 4 rectangles in different places on the coordinate grid. Label the vertices.

3. Look at the coordinates of the vertices for each rectangle. Look for patterns. Describe the patterns that you see, using words and symbols.

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