



Session 2306

# Designing Mathematics Professional Development Through Successive Approximations of Practice

NCSM 2017

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[Bridging.sri.com](http://Bridging.sri.com)

**SRI** Education



Images by:  
Verena Loewensberg



# Successive approximations of practice...

As a lens for PD activities

How we use the idea in our Bridging PD to meet our goals

How you can use it in designing your own PD activities



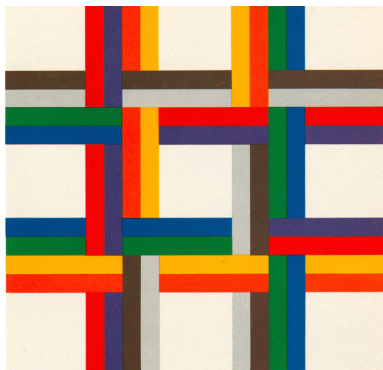
A little context: Bridging is professional development for mathematical argumentation in middle school.

Urban districts  
with culturally  
and linguistically  
diverse student  
populations

Summer institutes  
and  
School year:  
3 hrs/month  
alternating virtual  
and face to face

New book for teachers  
is based on Bridging.

Coming  
September  
2017



Includes PD guide  
with PD activities  
from this  
presentation

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# Mathematical Argumentation in Middle School

The What, Why, and How

A Step-by-Step Guide with  
Activities, Games, and  
Lesson Planning Tools

**CM** CORWIN  
MATHEMATICS

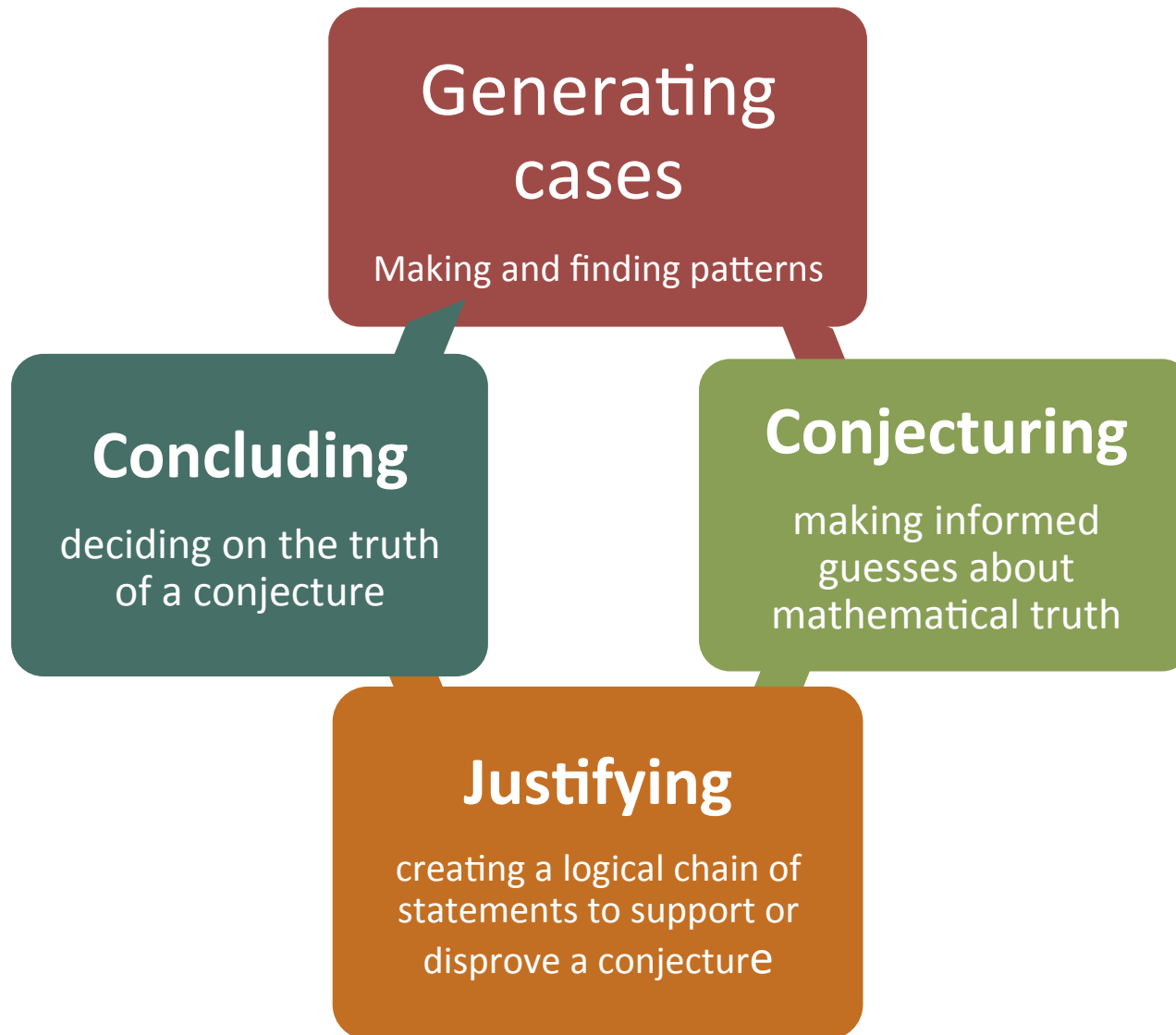
Foundational principle for Bridging:  
Teaching is disciplined improvisation!

“Good teaching is always improvisational... Teacher improvisations are always guided by structures that are important to effective teaching.”

—Sawyer, 2015

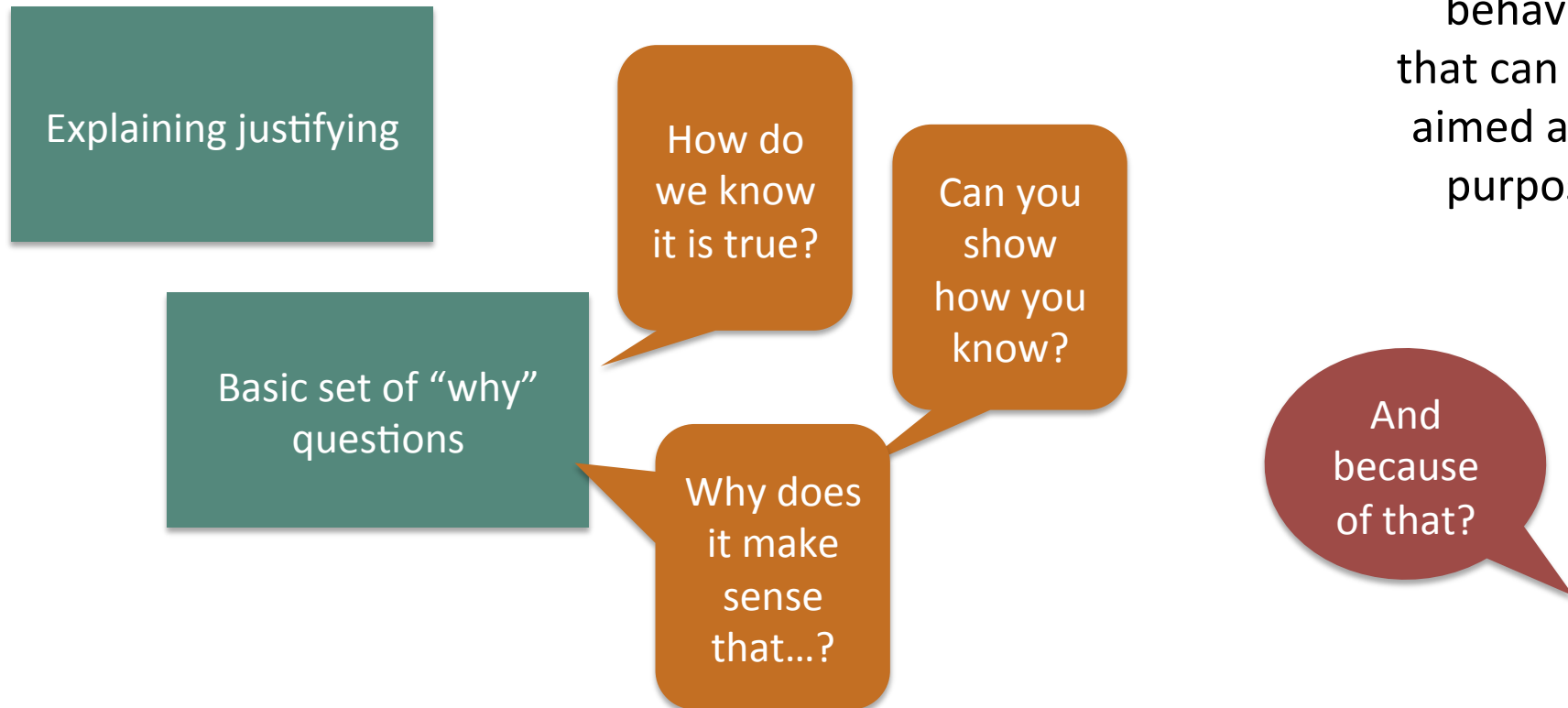


The *discipline* includes knowledge of the structure and nature of argumentation.



*Improvisation* is drawing on a set of teaching moves in the moment to support students' arguments.

**Moves**—the smallest piece of behavior that can be aimed at a purpose.



Our PD design problem:

If teaching is disciplined improvisation, how do we help teachers learn to **improvise** in the classroom, calling on their **knowledge** of argumentation?



We came to view PD activities through these lenses:

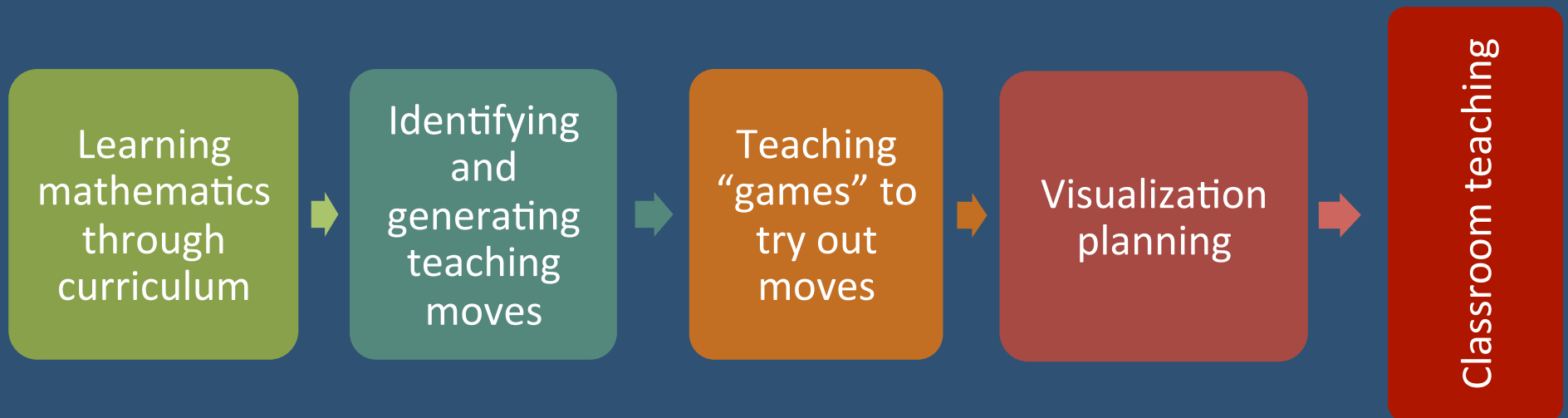


“A representation illustrates a facet of practice.”

“An approximation engages [teachers] in that practice.”

—Grossman, et al., 2009

Bridging activities that are **successive approximations of practice** get ever closer to improvisation in the classroom.



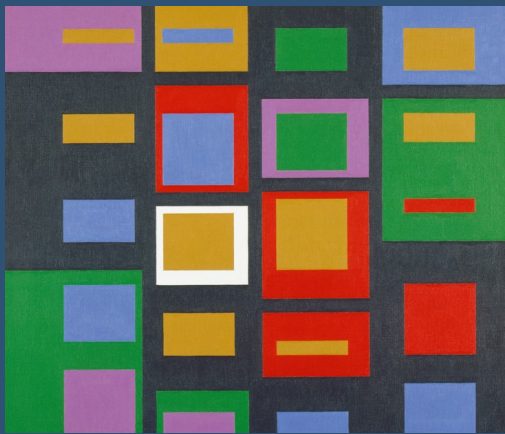
Learning  
mathematics  
through  
curriculum

Identifying  
and  
generating  
teaching  
moves

Teaching  
“games”  
to try  
out  
moves

Visualization  
planning

Classroom teaching



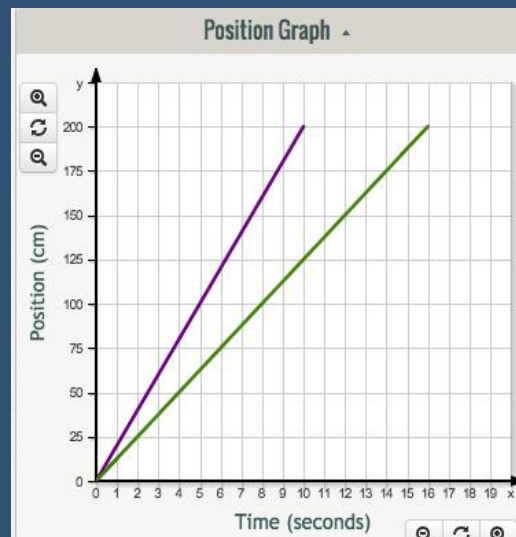
# In learning math through curriculum, teachers experience...

making bold  
conjectures

constructing  
justifications  
with others

exercising  
math  
authority

*Raj says that if a  
line is steeper  
than another,  
then it represents  
a faster motion.  
Is this always  
true?*



Learning  
mathematics  
through  
curriculum

allows...

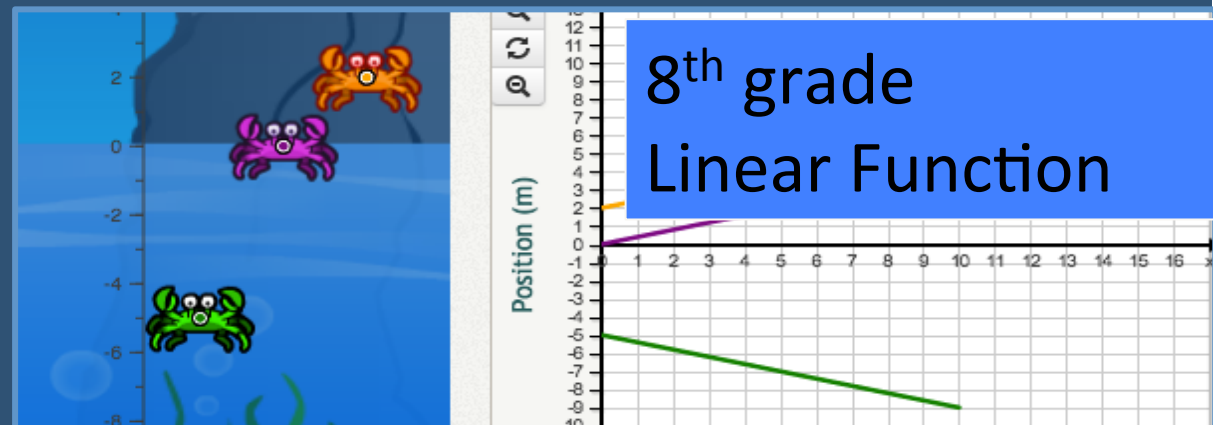
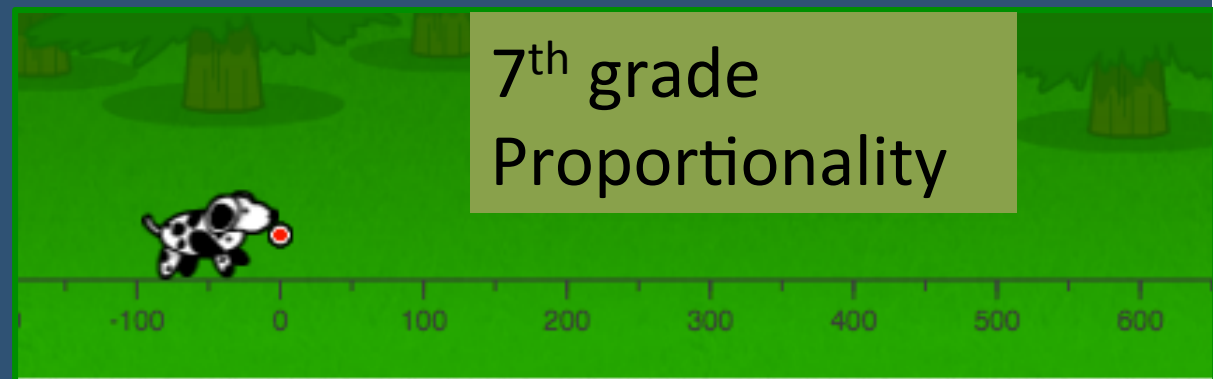
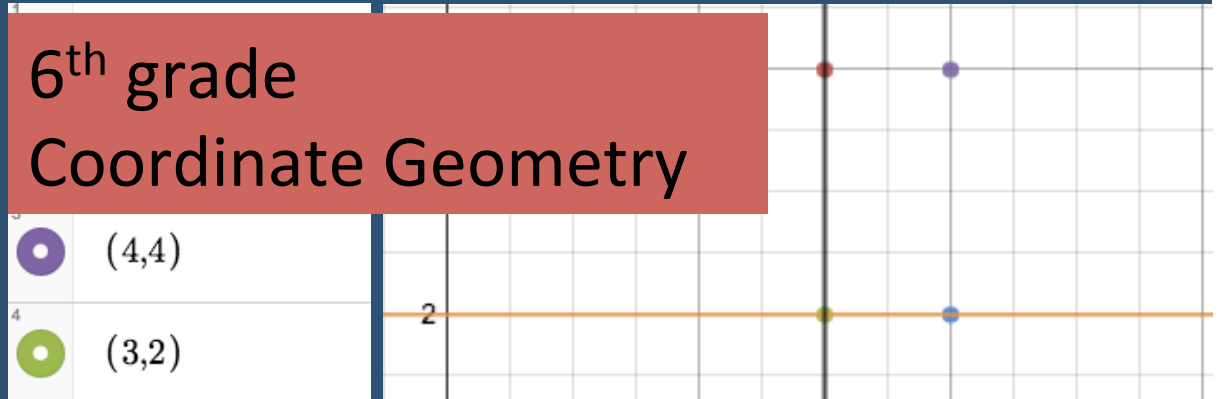
- teachers to experience argumentation
- leaders to model teaching argumentation

It provides a representation of practice.

Digital tools support argumentation.

Important: Teachers use with students.

See [bridging.sri.com](http://bridging.sri.com) for an example in GeoGebra.

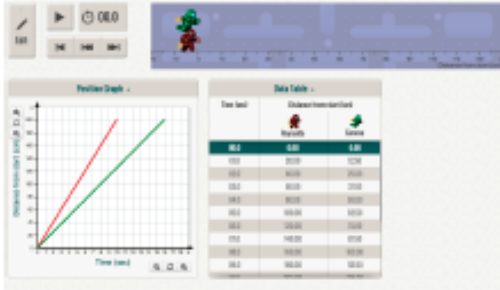


## Main Activity

30 min

### Activity 2.1

...for use  
back in the  
classroom.

		
Generating Cases	<p>A. Predict which robot will finish the race first. Press Play to check if you were right. Explain how you know.</p>	<p>Possible student answers: The red robot will finish first, because it arrives before the green robot; the red robot takes less time. In the explanation, elicit connections with the different representations.</p>
	<p>B. Which robot is going faster? Change the speed of the green robot. How did you do it?</p>	<p>Focus is on <b>speed</b>: Students can change the steepness of the graph to change the speed. They may also edit the starting position of the robot in the graph but this does not change the speed. And they can change the coefficient of <math>x</math> in the equation.</p>
	<p>C. Make one robot go really fast and the other robot go really slow. Press Play to observe the relationship between the robots' motions and their graphs.</p>	<p>Have students focus on what aspect of the graphs they have to change to modify the speed.</p>
Conjecturing	<p>D. Make a <b>conjecture</b> about what's always true about how the graph shows the speed of the robot.</p>	<p>Have students make conjectures in small groups. You may want to provide sentence starters to scaffold conjecturing. For example: If the green robot moves faster, then its graph....</p>

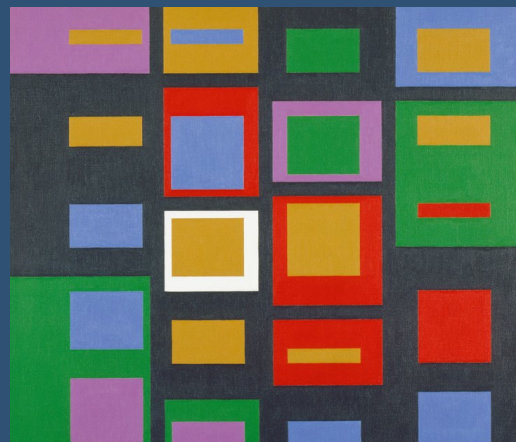
Learning mathematics through curriculum

Identifying and generating teaching moves

Teaching "games" to try out moves

Visualization planning

Classroom teaching





Enacting vignettes helps teachers *Identify* teaching moves—and purposes.

Teachers take parts and read aloud the classroom vignettes.

Leader asks

- **What move** did this teacher make in this line?
- What could have been her **purpose**?
- What was this vignette an example of?

Moves  
and  
purposes  
go  
together

# Sample from vignette

Ms. Cooper: So to say it a little more precisely: in the rectangle, the x-coordinates of the vertices on one of the vertical sides are both 4. Is that what you mean?

Ying: Yes, I think so.

Ms. Cooper: OK are we done? Have we justified the conjecture?

Bailey (pointing to all four rectangles): Yes, we see it right there.

Camden: The conjecture is supposed to work for *all* vertical sides. Maybe there could be one where it doesn't work.

Ms. Cooper: That's a good insight, Camden, that we need a justification that works for *any* vertical side of a rectangle. Who thinks they know?

Based on curriculum experience, identify and *generate* moves good for...



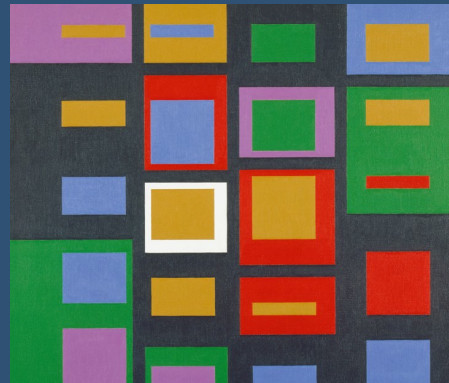
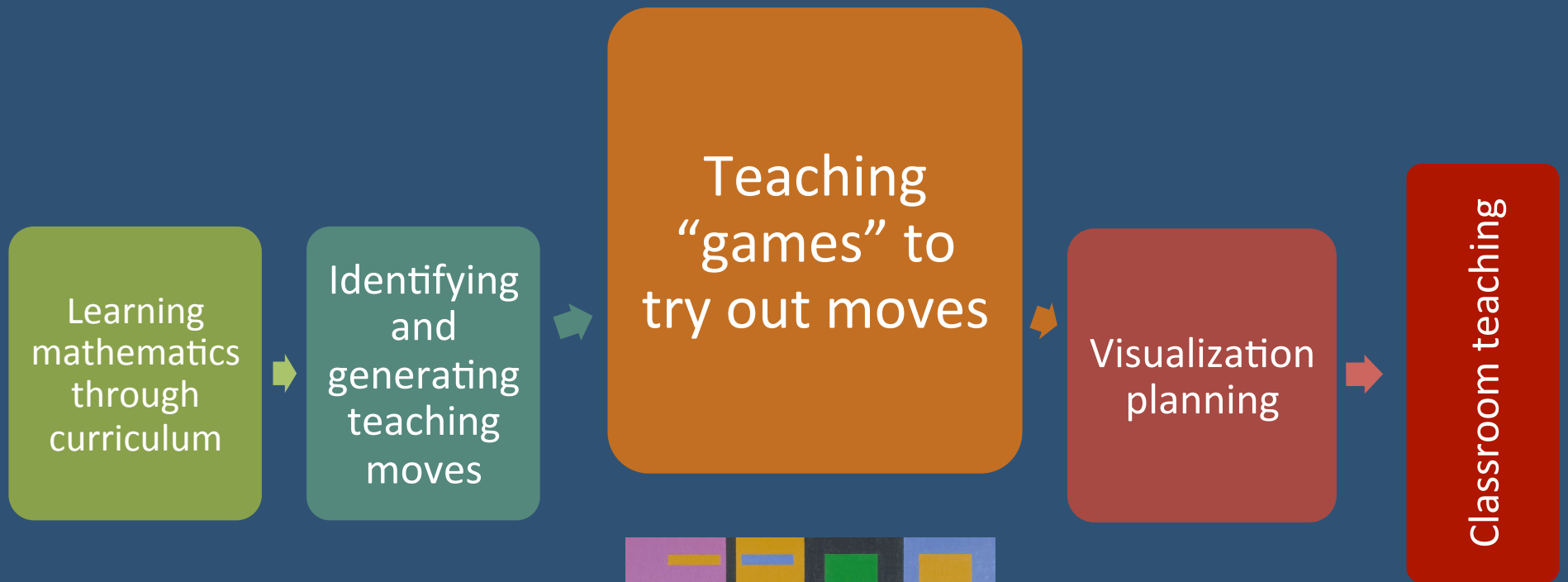
Teachers generate moves in a Google doc where they can share and organize them.

Identifying  
and  
generating  
teaching  
moves...

gets closer to practice because  
teachers *begin* to envision themselves as  
users of the moves.

Could you use these  
activities?  
What would they be like?  
What else could you  
design, based on these  
ideas?





Teaching games are based on improv games.

How improvisational actors learn their craft

NOT “anything goes”  
—rules structure  
open-ended participation

Freestyling is a kind of improv

Film all about improv



Lin-Manuel Miranda at the White House

# Applied improv is used in other fields.

## Alan Alda Center for Communicating Science



AT STONY BROOK UNIVERSITY

### Improvisation for Scientists

This innovative program is shepherded by Alan Alda, a founding member of the Alan Alda Center for Communicating Science. Joining Mr. Alda are Valeri Lantz-Gefroh, lead improv teacher for the Alda Center, Louisa Johnson, M.F.A. from NYU, and three members of Stony Brook University's Theatre Arts faculty, Lydia Franco-Hodges, Deborah Mayo and Steve Marsh.

The goal of teaching scientists improv is not to turn them into actors, but to free them to talk about their work more spontaneously and directly, to pay dynamic attention to their listeners and to connect personally with their audience.



NY Times  
March 29,  
2017



# Teaching “games” have a set of rules that focus on purpose.

Rules define...

- what the teacher should accomplish.
- the set of moves to use.

Teachers must play the game—a role play— not discuss it.

Post-game discussion: what moves worked for what purposes?



# Example teaching game

Round 1: closed-ended questions only

Round 2: open-ended questions only

1 teacher

2 students

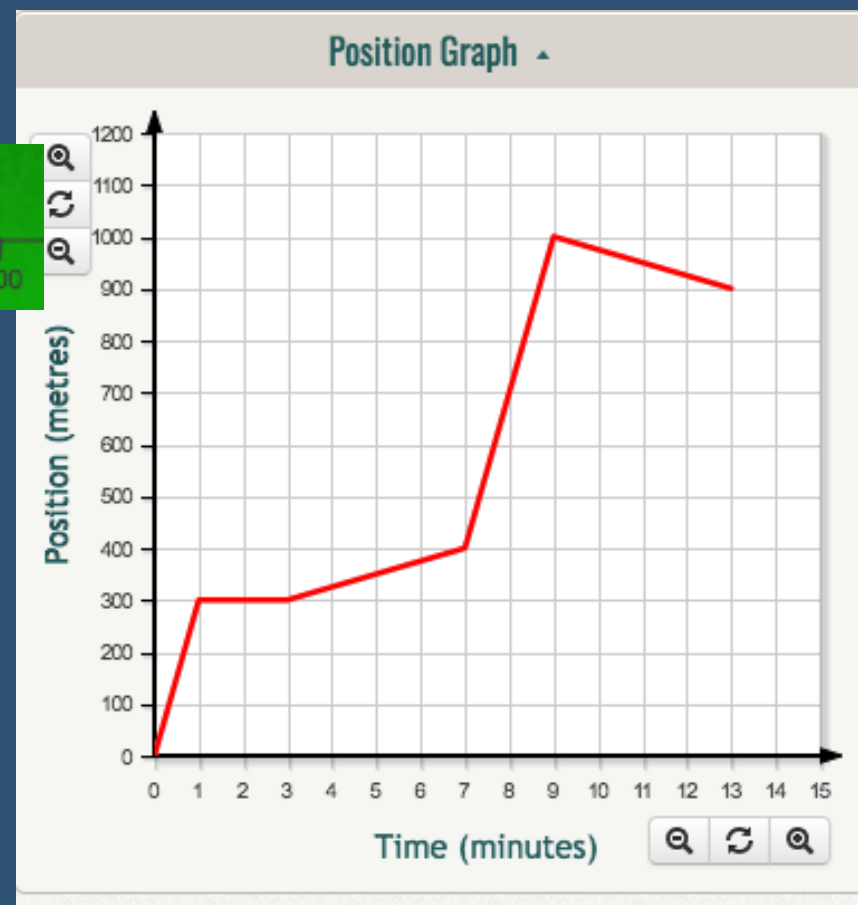
Facilitate justifying



Student conjecture

*When the line is flat, the dog is not moving.*

Reflect: What is each type of question good for?



## Teaching games...

move closer to practice because actual moves are enacted, purpose analyzed.

The teacher can deploy these in the classroom in a similar situation.

Could you use teaching games?  
What would they be like?  
What else could you design, based on this?



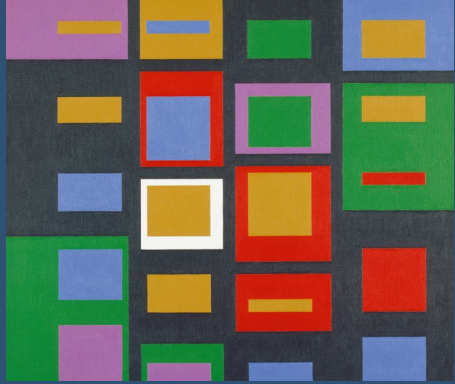
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Classroom teaching



# Visualization planning helps teachers imagine moves and responses.

1. Sit with another teacher.
2. Start with written lesson plan. Choose 10 min. segment.
3. Visualize:
  - One teacher **imagines the lesson outloud**, in detail, in order.
  - Listener records in the lesson plan.

- ✓ Focus on both moves and possible student responses.
- ✓ Follow possible paths.

Have also done between PD facilitator and teacher

# Visualization planning...

helps teachers develop an effective lesson planning practice.

relies on research about expert vs novice teachers.

makes explicit the expert's planning practice.

# Lesson plan form

**Task:** Students justify three conjectures

**Time:** 20 minutes

**Grouping:** Whole class

**Teaching moves**

Introduce the 1<sup>st</sup> conjecture. Elicit justification.

Show 4 rectangles on the display:

“typical”, square, pos and neg coordinates

“How do we know this *might* be true?”

**Students may:**

Use the typical rectangle as an example, stop there.

Do we know if it is always true based on that one example? How can we show it is true for all the rectangles?

use parallel and  $x = 0$  on  $y$ -axis

Ask what parallel means

Say same distance apart  
Never intersect

If needed:  
What does that tell is about coordinates?  
What does the  $x$ -coordinate mean?

Say  $x$ -coordinate tells distance from  $y$ -axis, so has to be the same on at the vertices of vertical lines

# Visualization dialogue (fictionalized)

**Ms. Cooper:** Yesterday, students made about 10 conjectures. I picked three of them to justify tomorrow. There's one about coordinates on vertical lines, and a similar conjecture about horizontal lines, but that second one is false—they got the coordinates backward. Then I picked the conjecture that I thought was a bridge to where I want to get to—the students using algebra to name the pattern of coordinates in any rectangle. It just states that there is a pattern across the 4 vertices, but doesn't state what that pattern is.

**Mr. Flores:** So, three conjectures that build to your algebraic conjecture, your “secret mission”? I see them here in your plan.

**Ms. Cooper:** Yes, and I think I'll start out with the true conjecture, just to ease them into argumentation. They aren't that experienced yet.

## Visualization planning...

moves closer to teaching practice as teachers engage in effective planning and envision their own classroom.

It provides a written record that teachers can use in the classroom.

Could you use  
visualization?  
What would it be like?  
What else could you  
design, based on this?







Classroom  
teaching

Teacher OK. So why are you saying this has to be true?

Student1 Because these ones, whenever we did them and it was greater than 180, these two didn't meet.

Teacher OK, but how do you know besides what you just did for a couple? Because there's a bunch of combinations you didn't do, right?

Student2 Uhhh....You said it was.

Teacher Did I? But how do I know?

Student2 Because that's the answer.

Teacher But why?

Student1 Because...

Teacher Because you gotta be able to back it up.

Student1 Because it makes sense.

Student2 Because with triangles, the two angles let the lines come together.

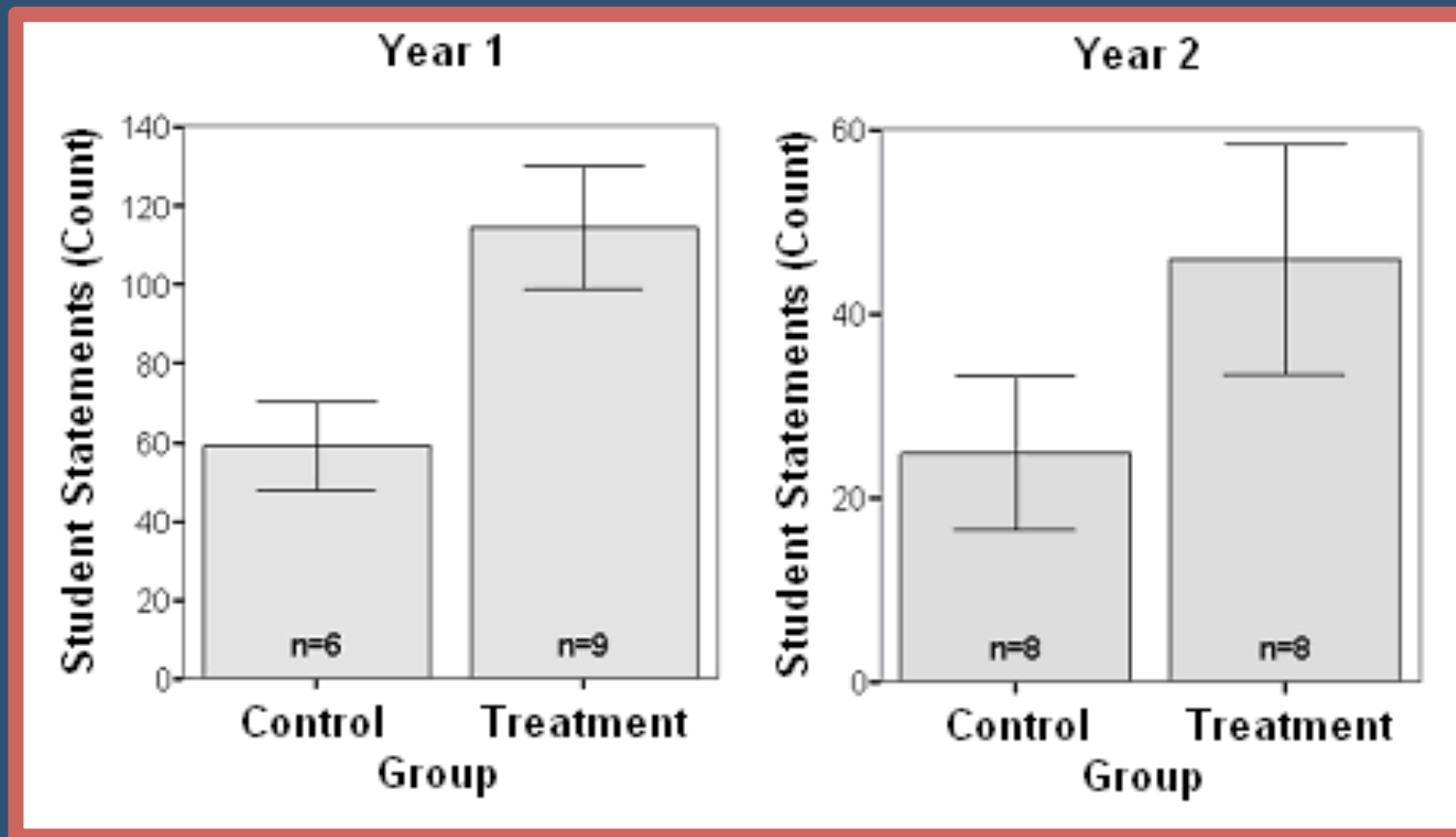
Teacher OK and if we use that logic, if 2 angles add up to more than 180, what would that mean?

Student1 That it won't meet with the other angle, it wouldn't be a triangle.

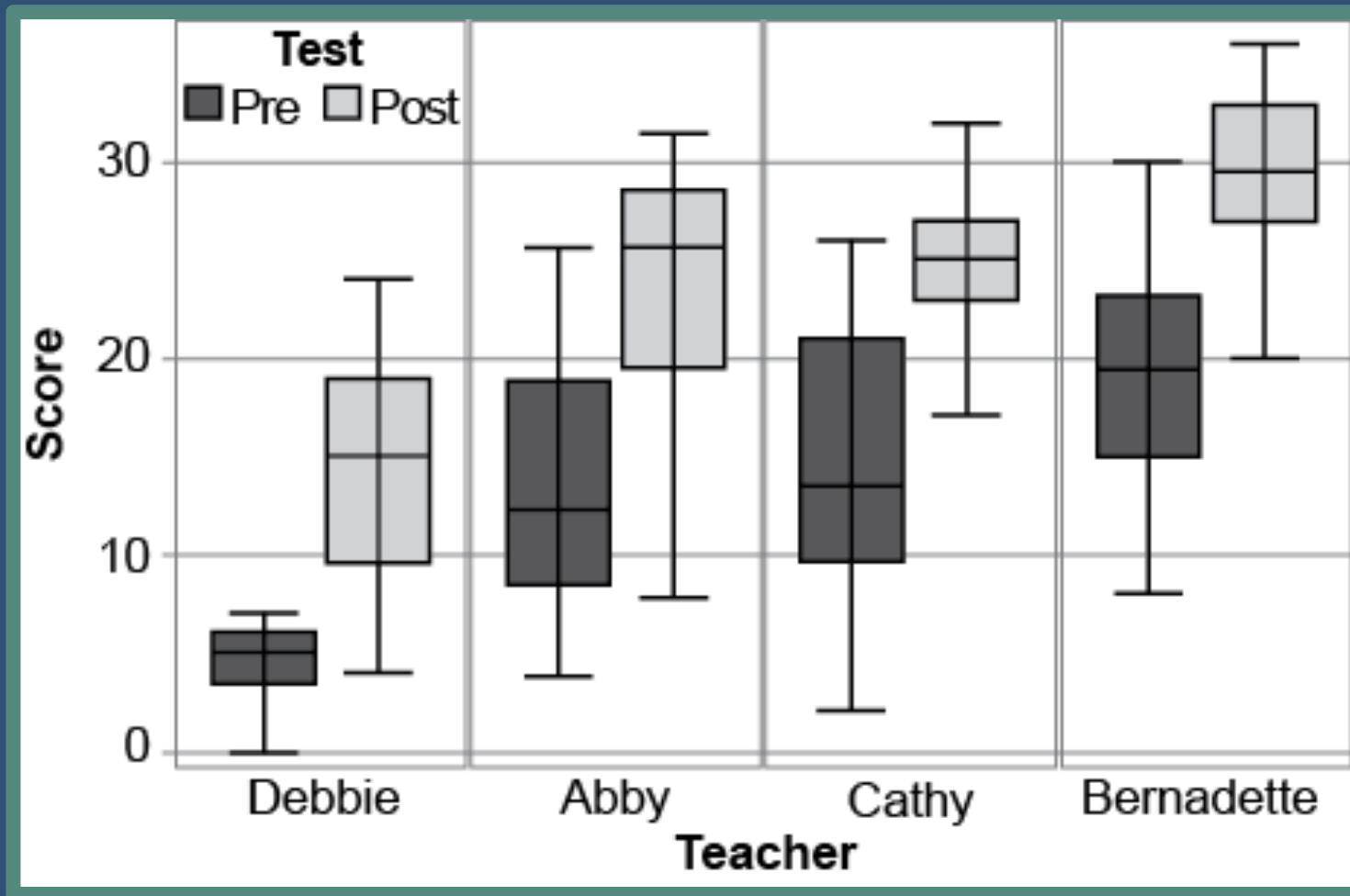
Teacher OK, write that down.

Using why questions repeatedly when students are beginning to argue

# Study 1: twice as much classroom argumentation



# Study 2: gain of 10 out of 36 pts in student learning



# We've learned some lessons.

Don't skimp on the mathematics learning.

Providing structure all along the way enables exploration in the right territory.

Focus on moves, purposes AND possible student responses.

# Designing approximations of practice

1. What are your PD goals?
2. How can we view your existing activities through the lenses of approximations and representations?
3. What would be a new activity, and how does it approximate and/or represent practice?
4. What two-way representations could bridge between PD and the classroom?

Learning  
mathematics  
through  
curriculum



Identifying and  
generating  
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Teaching  
“games” to  
try out moves



Visualization  
planning

