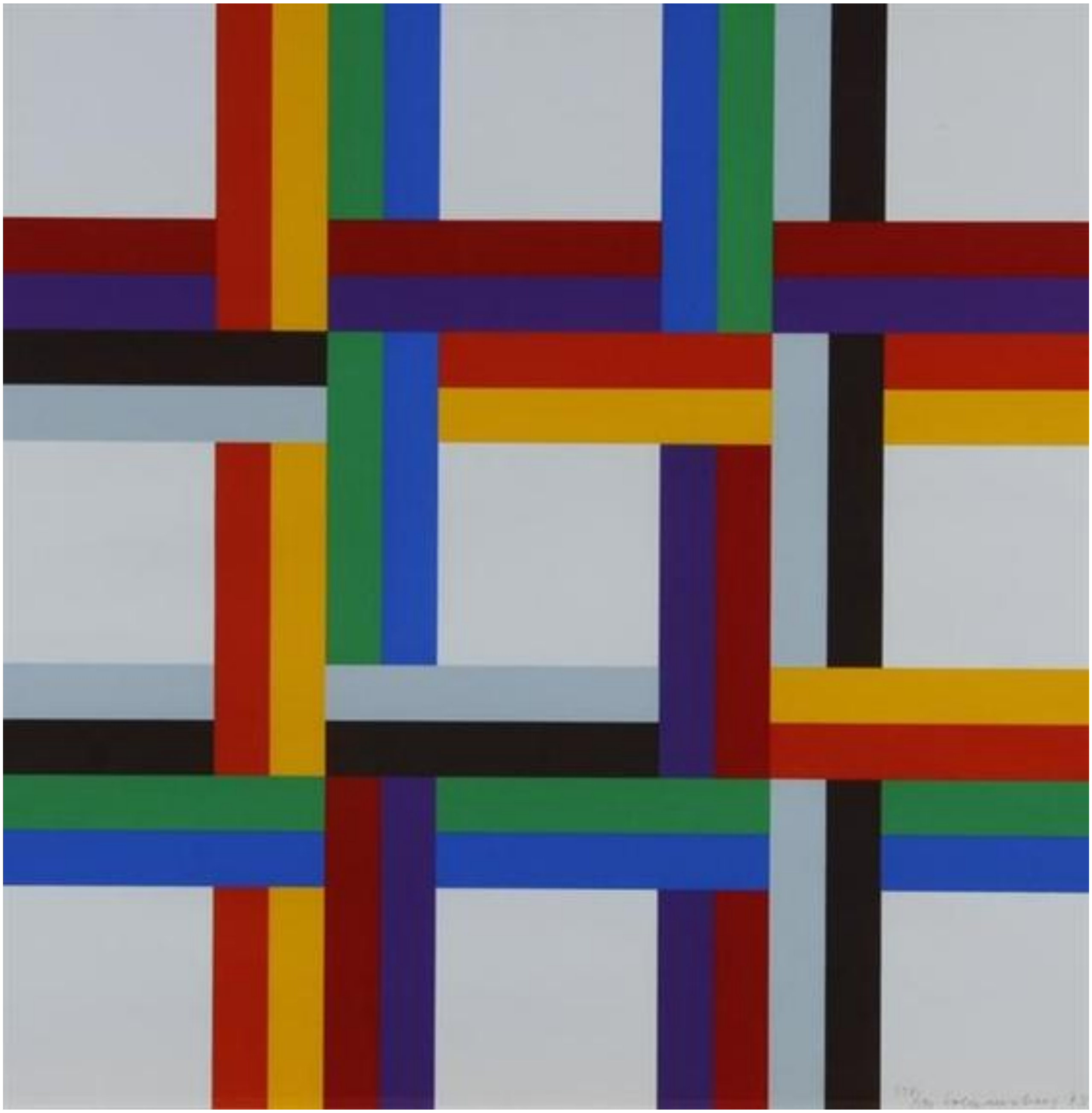

6G: Coordinate Geometry

http://bridgingmath.com

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6th Grade Coordinate Geometry

On the cover

Untitled, 1973. Verena Lowensberg.

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professional development

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Learning goals

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2 Patterns in Coordinates	How can rectangles be represented in a coordinate system?	Students will ● create polygons given coordinates ● generate conjectures about the relationships between coordinates in a rectangles with sides parallel to the axes, and justify them	6.G. 3.
3 Lengths of a segment	How do you find the length of a line segment in a coordinate system?	Students will ● generate conjectures about how to calculate the lengths of sides of rectangles with sides parallel to the axes, using the coordinates of the vertices	6.G. 3. 6.NS.6.c 6.NS.8

Justification Stages

1. Conjectures are justified/defended without evidence or by simply citing visual cues (because it looks like it), or relying on the teacher as the authority.
2. Conjectures are justified/defended by using one or two cases. This may include extreme cases—where the examples are the only evidence.
3. Conjectures are justified/defended by building on a case supported by generalized reasoning. This may start with an example, but then students develop a broader justification using the patterns that emerge in the example.
4. Conjectures are justified/defended with generalized mathematical reasoning that connects the conjecture and provides a rebuttal of counter-arguments. Counter-examples may be used.

Tips for Argumentation

Generating Cases

- To help students generate cases, tell them to create more than one example.
- To help students see general patterns, ensure that they look at a range of typical cases.
- To support students examining a wide range of cases, ask students to look at special cases (e.g, squares are special rectangles.)

Conjecturing

- To encourage students to generate conjectures, including bold conjectures for which the truth is unknown, ask
  - What are you pretty sure is true?
  - What do you think is true?
  - What do you think might be true?
  - What do you think is false?
- To encourage students to make a generalized conjecture, ask,
  - What do your cases have in common?
  - Can you make a conjecture that applies to more than one example?
- To encourage more mathematical conjectures, focus students' attention on the relationships between representations such as numbers, graphs, equations.

Justifying

- To start the justification process, ask students,
  - Is your conjecture always true?
  - What makes you think so?
- To encourage using visual representations to support language, suggest, “Show how you know it is true.”
- To advance justification, encourage students to think about extreme cases that will challenge their conviction about a conjecture.
- To challenge a justification, encourage students to think of a case that might not work.
- To advance from an example-based justification to a generalized one, ask students “how do you know your conjecture works in ANY case, beyond those you have used so far?”

Concluding

- To aid students in drawing a conclusion, ask them to summarize their arguments and write down what they know is true or false as a result.
- To provide closure, stamp or label a conjecture as “true”, “false” or “not enough information to decide.”
- To lead to the next argument, ask for new conjectures that follow from what they already have justified.

Lesson 1: Exactly Where?

Background

Essential Question

What is the coordinate system and how is it used to represent mathematical figures?

Learning Objectives

Students will

- understand the utility of the coordinate grid for locating points
- know the conventions of graphing in the coordinate plane

Common Core Alignment

5.G.1. Graph points on the coordinate plane to solve real-world and mathematical problem; Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate.

Warm Up 15 min

1. Introduce mathematical talk and norms for argumentation.
2. Introduce improv as a way to focus on those norms.
3. Play Zip Zap Zop.

Purposes: Make mistakes, pay attention, say yes, keep the ball in the air, make the ensemble look good, be present.

Norms developed: Pay attention, listen, make eye contact, make sure you are heard, participate.

Instructions:

- Stand in a circle.
- In turn, each person makes a verbal offer to another person by throwing him/her an invisible ball, saying *zip*, *zap*, or *zop* (one each, in that order).
- The catcher then becomes the thrower and sends the next word to someone else in the circle.
- Play continues in any order.

Questions for discussion about norms:

- What was it like to play that game?
- What did you learn about the Classroom Rights and Requirements?

Main Activity 30 min

Activity 1.1. Without a grid

	1. Sit with a partner so that you cannot see each other's work. A. On a piece of blank paper, draw a line segment of any length in any position. B. One person describes how to duplicate his/her drawing. C. The other follows the instructions. • Do not use numbers. • Do not use units such as inches, centimeters. • Do not draw axes.	*pair work* Make sure students do not use standard units or measurement tools. Encourage them to be as creative as possible in communicating about their drawings to their partners.
	2. When you are done, compare your drawings: A. Are your line segments congruent? B. Are they placed in the same location on each page? C. How do you know?	*whole class* Ask questions: • Why are the drawings the same or not? • What words did you use to help communicate your drawing? Collect on the board the words students used to describe position, length, and slant of their line segment.

Activity 1.2. With a grid

	1. Switch the roles with your partner. A. On a piece of grid paper, draw a line segment of any length, in any position. B. One person describes how to duplicate his/her drawing. C. The other follows the instructions. • Use the grid to help you. This means you can use numbers to counts the boxes.	*pair work* Make sure students do not use standard units or measurement tools. Encourage them to be as creative as possible in communicating about their drawings to their partners.
	2. When you are done, compare your drawings: A. Are your lines congruent and in the same place? B. How do you know?	*whole class* Have students compare the tasks with and without the grid. • How did you use the grid to explain to your partner how to create the line segment? • How was this task different or similar to the task without the grid? • Was it easy or difficult to communicate with your partner compared to the task without grid? why?

	3. If time allows, switch roles and repeat, this time drawing axes on the grid.	If needed, review vocabulary for coordinate plane; axes, origin, coordinates, quadrants.

Lesson 1: Homework

1. You are the commander in a control tower. You have to indicate where rocks are so that sail boats don't run into them. Help them to travel safely by giving them the coordinates of the rocks.

Point A: (,   )

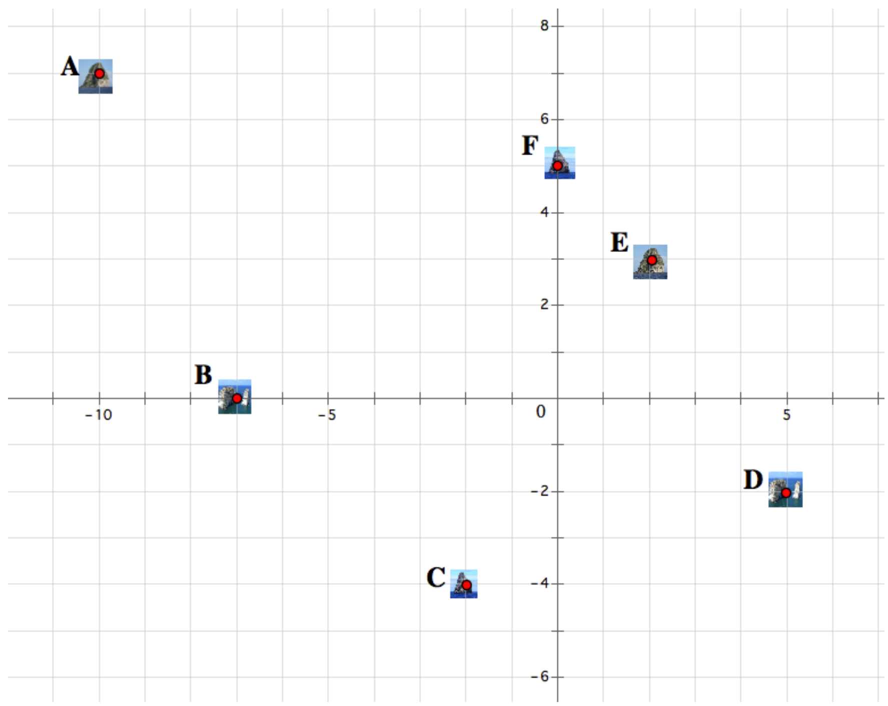
Point B: (,   )

Point C: (,   )

Point D: (,   )

Point E: (,   )

Point F: (,   )



Lesson 2: Patterns in Coordinates

Background

Essential Question

How can rectangles be represented using a coordinate system?

Learning Objectives

Students will

- generate conjectures about the relationships between coordinates in rectangles with sides parallel to the axes and justify them.

Common Core Alignment

6.G. 3. **Draw polygons in the coordinate plane given coordinates for the vertices**; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Warm Up 10 min

<table>
<tr>
<td>Gift Giving
</td>
<td>See ppt.

Purpose: Making offers, accepting offers regardless of their clarity, making your partner and the ensemble look good, saying yes.

Norms developed: Listening and building off of each other's ideas.

Instructions:
• Partners stand facing each other with a huge closet of unlimited gifts behind them.
• One partner offers the other a gift from the closet, by handing him/her a wrapped box. This gift can be of any dimension, and the offerer gives an idea of the size, weight, and/or shape.
• The receiver then opens the gift and names it by thanking the giver (e.g., "Thank you for this grapefruit") as he/she picks up and handles the gift.
• The giver then responds with how he/she selected the gift and why he/she knew the receiver would enjoy it. The roles are then switched.
Questions for discussion about norms:
• What was it like to play that game?
• What did you learn about the Classroom Rights and Requirements?
• What did you learn about making conjectures? (i.e., Conjectures are like gifts.)</td>
</tr>
</table>

Main Activity 30 min

Activity 2.1.

Generating cases	1. Drag vertices A, B, and C of the red rectangle to make rectangles of different sizes and at different locations. a. What seems to happen to the coordinates as you drag each vertex? b. Make the following rectangles. Record the coordinates of their vertices in your notebooks. • A long, skinny rectangle • A square • A rectangle with vertex (0, 0) • A rectangle that is in two quadrants of the coordinate grid	*group work* • Have students drag each point one at a time and make observations about how dragging each point changes the rectangle. • Moving point D does nothing to the rectangle. • Make sure students drag points beyond Quadrant 1. • Make sure students drag points one at a time and observe how the coordinates are changing. • Encourage students to start writing the patterns they observe. • Provide a table so that students can organize coordinates of each rectangle • Ask students to observe what changes and what doesn't change in coordinates as they drag vertices.
	2. Describe **patterns** that you see in the coordinates of the vertices.	• Have students focus on the coordinates of the vertices. • Check student understanding of the term "vertex."
Conjecturing	3. Make **conjectures** about the relationships between the coordinates of the vertices of all rectangles with sides parallel to the *x* and *y*-axis.	*whole class* Elicit student conjectures: • What do you mean by this particular word? Is there another way to say it, mathematically? • What do we already know for sure? • Do you think that the pattern holds for any case you could generate? If so, how can we describe that, mathematically? If not, try to describe just the cases for which it does hold. • Can you use more mathematically precise language for stating the pattern? Build on student conjectures: • What else is true about the coordinates of the vertices? • What repeats and what changes as you go from vertex to vertex?

Justifying	4. **Justify** one of your conjectures using what you know about rectangles and the coordinate grid, and what you can see when you make changes with the red rectangle.	*whole class* Elicit student understanding • What is your definition of a rectangle? • Can a square be a rectangle? Can a rectangle be a square? • How did you use your definition to explain the patterns you observed from the coordinates? • What can you tell about the coordinates of points that are on a line parallel to *x*-axis? *y*-axis?
Conclusions	5. **Conclude**: What do we know about the coordinates of vertices of rectangles with sides parallel to the axes?	*whole class* • Mark conjectures true or false as you justify them. • Summarize the argument and make sure there is consensus about the truth or falsity of the conjectures. • Make sure that students know which conjectures are true and which are false (whether by consensus or other means). • **Encourage the students to write down their conclusions.**

Arguments

Conjecture	Justification	Conclusion
The *x*-coordinates are always the same on the same vertical side. Similarly, the *y*-coordinates are always the same on the same horizontal side.	If any two points are on the same vertical sides, their *x*-coordinates are the same because the side is parallel to the y-axis and the point on the same side is the same distance away from the y-axis.	True
You need 4 different numbers *at most* to fill in the coordinates of the vertices for any rectangle with sides parallel to the axes.	Coordinates of the two vertices of the same side have common *x*- or *y*-coordinates. You can have only 2 different *x*-coordinates and 2 different *y*-coordinates. Therefore, you need at most 4 different numbers to use to make coordinates pairs for the vertices of a rectangle.	True
You need only 2 numbers to make coordinates of the vertices to make a rectangle.	Sometimes you can use only 2 numbers to create a rectangle as shown in the example, but it is not necessarily true for any rectangle. Example. (1,1), (2,1), (1, 2), (2,2) - square.	False

If two vertices are the endpoints of the diagonal, they do not have a common *x*- or *y*-coordinates between the vertices.	Proof by contradiction: If they have common *x*- or *y*-coordinates, they would be on the same vertical or horizontal line. Therefore, they can't be on a diagonal. Note: Proofs by contradiction are not easy for students to construct and understand. Make clear the proof starts by contradicting the conclusion, i.e. "What if the vertices *did* have the common coordinates?"	True
If two points on a line have the same *x*-value, then the line is parallel to the *y*-axis. Similarly, if two points on a line have the same *y*-value, then the line is parallel to the *x*-axis.	If two points have the same x-coordinate but different y-coordinates, they will be lined up vertically. Therefore, the points will be on the line that is parallel to the y-axis.	True
Given two lines, if one line is parallel to the *y*-axis and the other line is parallel to the *x*-axis, the lines are also perpendicular to each other.	This is true because the axes are perpendicular to each other.	True

Lesson 2: Homework

For each set of coordinate pairs,

- Plot points using the coordinate pairs provided.
- Connect the points in the same order listed.
- What shape is it? Explain how you know.

A. (3,5), (3,7), (5,7), (5,5)

rectangle (square)

B. $(-2, 2), (-2,6), (-8,6), (-8,2)$

rectangle

C. $(-5,-3), (-4, -3), (-4, -7), (-5, -7)$

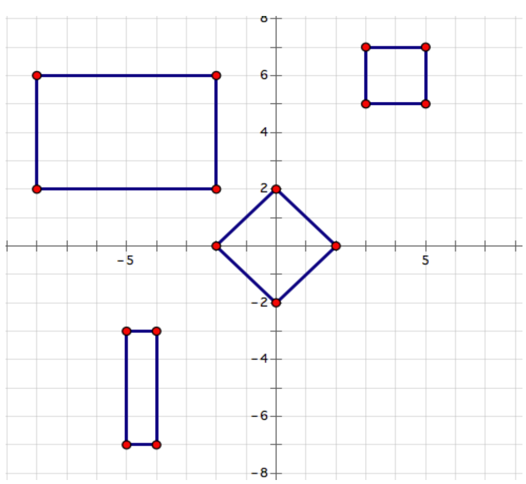
rectangle

D. $(0,2), (2,0), (0, -2), (-2,0)$

rectangle (square)

Teacher note: Provide grid paper with axes. Students may explain that they know a shape is a rectangle by visual inspection only. Press them to think about definitions of rectangles and how they can use grids to figure out angles. *d* would be a challenge to explain how there is a 90° angle at each vertex.

Answer:



Lesson 3: Lengths of segments

Background

Essential Question

How do you find the length of a line segment in a coordinate system?

Learning Objectives

Students will

- generate conjectures about a strategy to determine length of a segment using coordinate pairs.

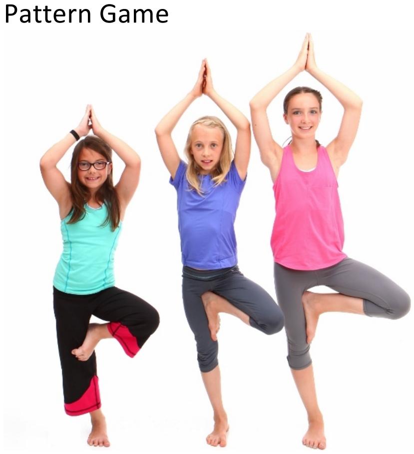
Common Core Alignment

6.G. 3. Draw polygons in the coordinate plane given coordinates for the vertices; **use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.** Apply these techniques in the context of solving real-world and mathematical problems.

6.NS.6.c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; **find and position pairs of integers and other rational numbers on a coordinate plane.**

6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Warm Up — 5 min

<table>
<tr>
<td>Pattern Game
</td>
<td>Play Pattern Game
Purposes: Make mistakes, pay attention, say yes, keep the ball in the air, make the ensemble look good, be present.
Norms developed: Pay attention, make bold conjectures, look for patterns, build off each other's ideas, participate.
Instructions:
• Players stand around the perimeter of the room, so that everyone can see everyone else. In the first round, tell students they'll make the pattern AABB. The first student (A) can do or say anything (e.g., reach hands up high). The second student would have to repeat the same motion. The third student would make a new pattern (B, e.g., moos like a cow). The fourth student would follow. The following students follow the pattern.
• For the second round, suggest an ABC pattern.
• Remember to use "circus bow" to .</td>
</tr>
<tr>
<td></td>
<td>Questions for discussion about norms:
• What was it like to play that game?
• What did you learn about the Classroom Rights and Requirements?</td>
</tr>
</table>

Main Activity 30 min

Activity 3.1

<table>
<tr>
<td rowspan="2">Generating cases</td>
<td>1. You are going to play a game. Use 1, 8, and 5 to make the coordinates of two endpoints of a line segment. The line segment must be parallel to either the x- or y-axis. The person who makes the longest segment between their two points wins.

Drag points A and B to make segments with the coordinate pairs you came up with. Check their lengths and see which coordinate pairs produce the longest segment.</td>
<td>groups of 4
Ask questions:
• How many numbers do you need to create coordinate pairs in order to make a line segment that is parallel to the axes? Why? (This should be a review from the previous lesson)
• What patterns do you see?
If needed, have students record their coordinates and their lengths in their notebook.</td>
</tr>
<tr>
<td>2. In Round 2, each group chooses any three positive whole numbers. This time, players can change the sign of one of the numbers. Play again to make the longest line segment.

Drag points in each segment to make segments with the coordinate pairs you came up with. Check their lengths and see which coordinate pairs produce the longest segment.</td>
<td>groups of 4
If needed, have students record their coordinates and lengths in their notebooks.</td>
</tr>
<tr>
<td>Conjecturing</td>
<td>3. Make conjectures:
A. Make a conjecture about how you can find the length of a line segment given 2 endpoints with their coordinates.
B. Make another conjecture about how you can always win the game.</td>
<td>groups of 4
Ask:
A. How did you use the grid? How can you express this in terms of operations? (a bit beyond grade-level standards)
B. How can you use absolute value to help you express your conjecture?</td>
</tr>
</table>

Justifying	4. **Justify** your conjectures.	*whole class or same groups* Ask questions: • How did you find the length of a segment using the coordinate pairs of the segment? • How did you select numbers to make the longest segment? • Which number did you decide to change sign? Why? How did that help lengthen your line? • How did you find the length of the segment when you had both positive and negative numbers as coordinates of the endpoints? • What is the length of the segment when you have positive numbers only as coordinates of the endpoints?
Concluding	5. Write down your **conclusion.**	*whole class* • Mark conjectures as true or false as you justify them. • Summarize the argument, making sure there is consensus about the truth or falsity of the conjectures.

Arguments, Part A

Conjecture	Justification	Conclusion
If a line segment is parallel to the *x*-axis and both the *x*-coordinates are positive, you get the length by subtracting the smaller x-coordinate from the larger x-coordinate.	If a line segment is parallel to the *x*-axis, their *y*-coordinates are the same. If both *x*-coordinates are positive, the segment is in Quadrant 1. The length is the difference between how far the larger *x*-coordinate and the smaller *x*-coordinate are from the *y*-axis.	True
If a line segment is parallel to the *y*-axis and both the *y*-coordinates are positive, you get the length by subtracting the smaller *y*-coordinate from the larger y-coordinate.	If a line segment is parallel to the *y*-axis, their *x*-coordinates are the same. If both y-coordinates are positive, the segment is in Quadrant 1. The length is the difference between how far the larger *y*-coordinate and the smaller *y*-coordinate are from the *x*-axis. Teacher Note: When there is a negative coordinate, one has to take into account absolute value – in other words, the negative coordinate’s distance from zero.	True

If a line segment is parallel to the *x*-axis and the *x*-coordinates have opposite signs, you get the length by adding the positive *x*-coordinate and the absolute value of the negative *x*-coordinate.	If *x*-coordinates have opposite signs, then the endpoints appear on the opposite sides of the *y*-axis, so the length of the segment is the sum of how far one point is from the *y*-axis and how far the other point is from the *y*-axis. Or, $5 + \|-3\| = 8$	True
If a line segment is parallel to the *y*-axis and the *y*-coordinates have opposite signs, you get the length by adding the positive *y*-coordinate and the absolute value of the negative *y*-coordinate.	If y-coordinates have opposite signs, then the endpoints appear at the opposite sides of the *x*-axis, so the length of the segment is the sum of how far one point is above the *x*-axis and how far the other point is below the *x*-axis.	True
For a line segment parallel to the *x*-axis, if both *x*-coordinates are negative, you get the length by adding *x*-coordinates and changing the sign.	If a line segment is parallel to the *x*-axis, then their *y*-coordinates are the same. If both *x*-coordinates are negative, the segment is in Quadrant 2 or 3. The length is the difference between how far left *x*-coordinates are from *y*-axis, or in other words, the distance between two coordinates.	True
You get the length of a segment by adding two numbers.	Counterexample. (1, 3), (4, 3). If you add 1 and 4, you get 5, but the length is 3 units. However, if you have (–1, 3) and (4, 3) and count how many units you move from 4 to –1, then you get 5 units, the length of the segment. The validity also depends on which two numbers one adds – you can't add *x*-coordinates to *y*-coordinates, but only corresponding *x*- or *y*-values. Teacher note: Ambiguity of the statement allows other possible combinations of two coordinates, resulting in rejection of the statement for its validity.	Not always true
You get the length by subtracting one number from the other.	Counterexample. (1, 1), (1, –5). If you do 1– (–5), then you get the length. On the other hand, if you do –5–1, then you get –6, which can't be the length because length can't be negative. Teacher note: Ambiguity of the statement allows other possible combinations of two coordinates, resulting in rejection of the statement for its validity. Teacher Note: Students do not need to do calculations with integers, but make sure to address why they want to subtract one number from another and what that means in terms of getting a distance between two points or length of a segment.	Not always true

You get the length by counting grids along the segment.	Counting the grids can sometimes work if one grid represents 1 unit, but it is not true in general.	Not always true

Argument, Part B

Conjecture	Justification	Conclusion
Pick the two largest of the numbers and change the sign of one of the numbers. Then use the two numbers as the *x*- or *y*-coordinates for both endpoints.	The length of a segment is determined by how far apart two points are. The x- and y-coordinates represent how far a point is from zero on the *x*- or *y*-axis. So if you have one positive and one negative number, the points are farthest apart. So I pick the two largest numbers (when both are positive) and change one of them to be negative. For example, given 2, 3, 5: ● I picked 3 and 5 and changed 5 to − 5, then I made (2, − 5) and (2, 3) to make a vertical segment. Its length is 8 units.	True
Use the smallest number as the x-coordinate and the two largest numbers as the y-coordinates without changing a sign.	Counterexample. Given 1, 2, 3: ● Using the rule, you create (1, 2) and (1, 3), which has length of 1. ● On the other hand, if you change 2 to –2, you can create a segment of length 5 (e.g. (1, –2) and (1, 3)).	False
Order the number from the smallest and the largest and pick the smallest and the largest without changing a sign.	Counterexample. Given 2, 3, 5: ● If you pick 2 and 5 without changing their sign, you can create a segment of length 3 (e.g. (2 3) and (5, 3) ● On the other hand, if you change 5 to –5, you can create a segment of length 8 with (2, 3) and (2, –5), which is longer than the segment of length 3.	False

Lesson 3: Homework

1. Each set of coordinate pairs represent the vertices of a square or a rectangle. For each set, determine if the vertices will form a square or rectangle WITHOUT drawing. Explain how you came up with the answer. Check if you are right by drawing.

   A. (5, 2), (5, 5), (9, 2), (9, 5)

   B. (–1, 3), (1, 3), (–1, 5), (1, 5)

   C. (–4, –7), (–1, –7), (–4, –4), (–1, –4)

   D. (2, 2), (2, -3), (3, 2), (3, -3)

Possible Answer: A, C, and D make rectangles and B makes a square. I noticed how far apart the *x*-coordinates were from each other as the *y*-coordinates were from each other. If the distances are the same, the vertices make a square, because in squares, all sides have the same length. In B, the *x*-coordinates (–1 and 1) are 2 units apart and the *y*-coordinates (3 and 5) are also 2 units apart, therefore, the coordinate pairs will form the vertices of a square with side lengths of 2 units.

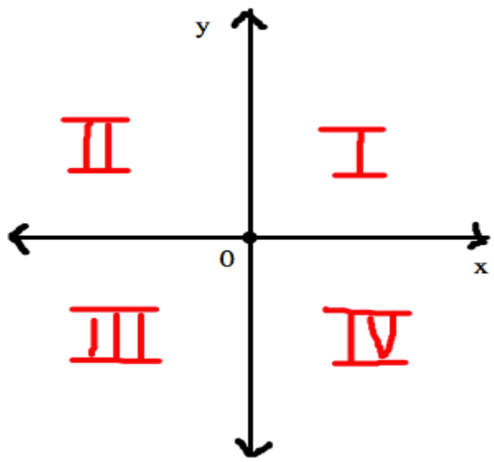
Glossary

Axes: a pair of perpendicular number lines.

Origin: the point where two axes intersect.

Coordinates: the ordered pairs of numbers. The first number tells how far to go in the direction of *x*-axis and the second number tells how far to go in the direction of *y*-axis from the origin. e.g. (2, 5) represents *x*-coordinate as 2 and *y*-coordinate as 5.

Quadrants: 4 sections of the plane divided by the *x*- and *y*- axes. They are numbered as I, II, III, IV, counterclockwise starting from top right section as shown below.



Absolute value: the distance from zero on the number line or the magnitude of a positive or negative quantity. e.g. $|-2| = 2$.