



Bridging.sri.com

Session 448

# Mathematical Argumentation Lessons: Engaging All Students

NCTM 2017

Harriette S. Stevens | Jennifer Knudsen | Teresa Lara-Meloy

**SRI** Education



Images by:  
Verena Loewensberg





# Bridging is professional development for mathematical argumentation in middle schools.

Urban schools with culturally and linguistically diverse student populations

Summer institutes  
AND  
School year:  
3 hrs/month  
alternating virtual  
and face-to-face

Interactive digital curriculum units  
AND  
adaptations of adopted curriculum



## In this session

- Characteristics of learning environments supportive of diverse student groups, particularly in urban schools
- Improvisational activities that promote student identity and empower learners
- Mathematical argumentation and curricular activities that can enrich learning environments
- Discussion

## Bridging PD: We work in partnership with large urban school districts.

- High proportions of under-served and economically disadvantaged students
  - African American/Black youth
  - Latin@ youth
  - English Language Learners
  - Over 50% free and reduced price lunch
- These youth often lack access to, or the opportunity to learn, higher-level mathematics.



Research informs our work and provides insight into characteristics of learning environments important to specific student groups common in urban settings, in particular:



## Characteristics (among others) of learning environments important to African American/ Black youth

- High-energy classrooms
- Sustained interaction with others
- Majority of learning in communal, group settings



## Characteristics (among others) of learning environments important to Latin@ youth

- Beliefs that one is competent and in control of one's own learning
- Perceptions of receiving support from others, including peers and teachers



## Characteristics (among others) of learning environments important to English learners

- Recognition of all of the resources and experiences students bring to the classroom, including their first language
- Use of multiple representations for participation, including gesture





## High expectations: Important for *all* students

- Academic rigor
- Use of high-level vocabulary
- Conceptually–rich mathematics content
- Focus on mathematical practices, not just practicing math skills

# So why argumentation?

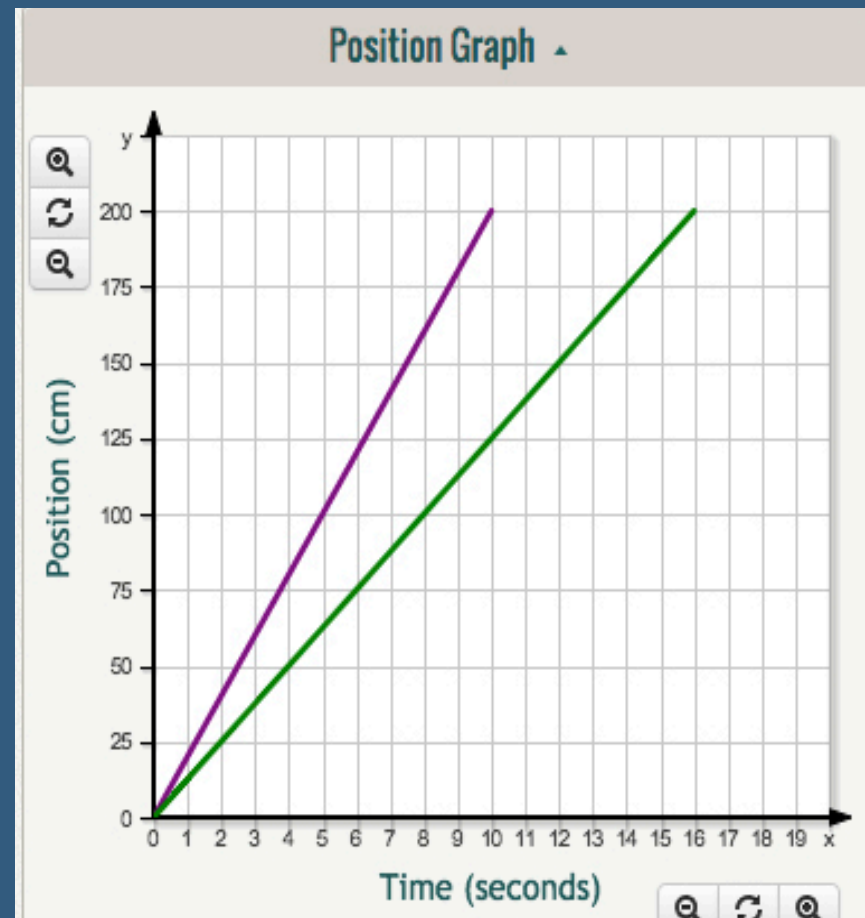
Common Core MP 3:  
*Construct viable  
arguments & critique  
the reasoning of others.*

High-level disciplinary  
practice: what  
mathematicians do

Fosters conceptual  
understanding

Advances 21<sup>st</sup> century  
workplace skills

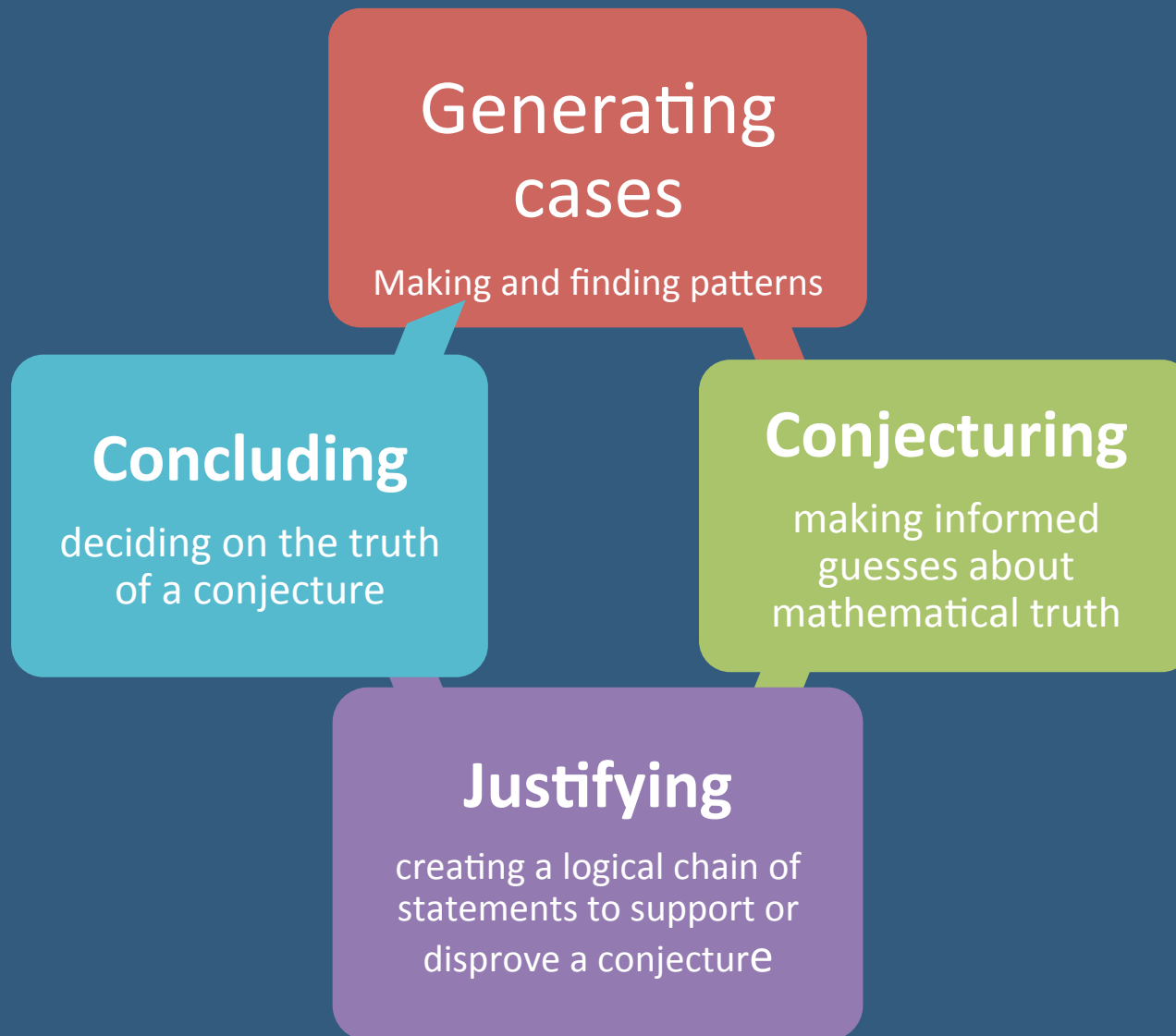
Argumentation is about deciding what's true or false in a mathematical situation.



Amani says: "If a line is steeper than another, then it represents a faster motion."

Is this always true?

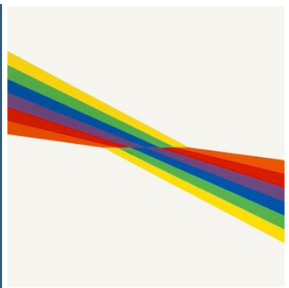
# Our view of argumentation in four parts



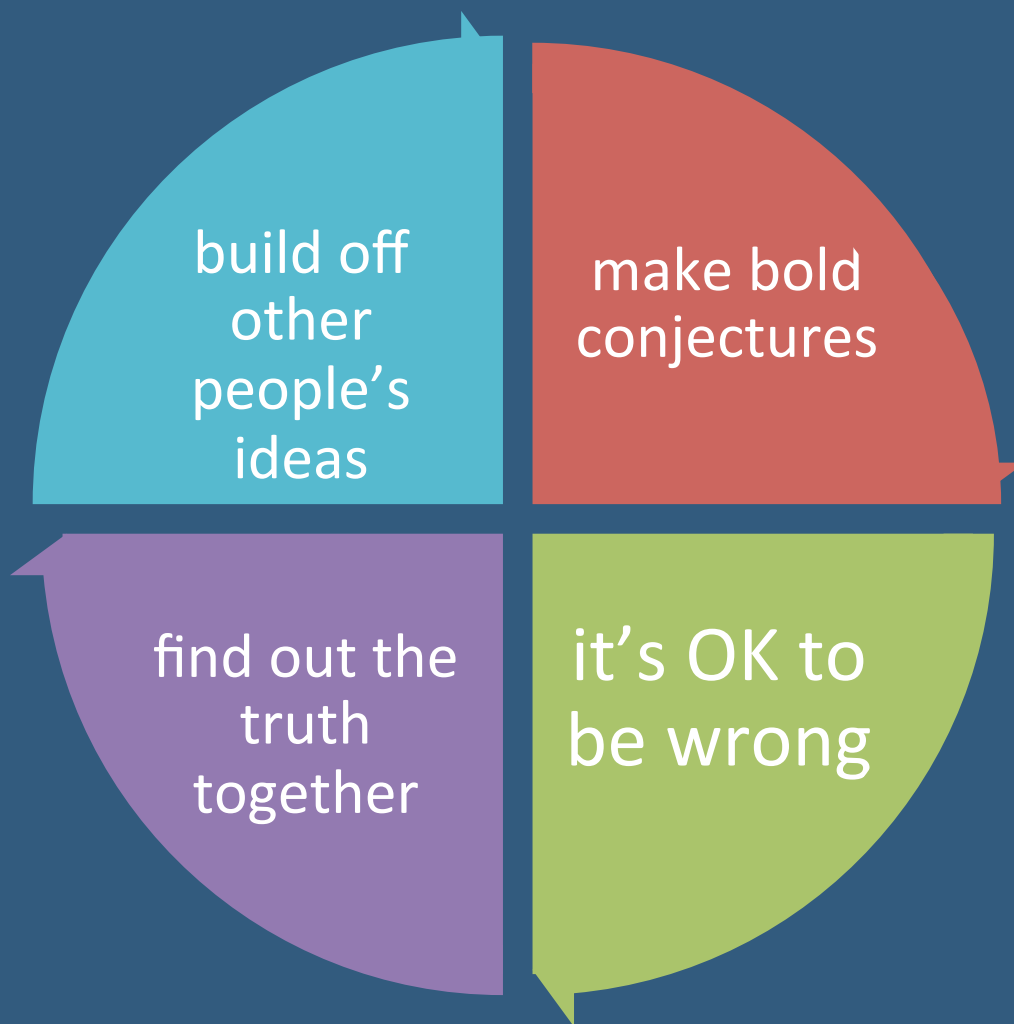


## Students become mathematical truth seekers.

- Argumentation is a social practice.
- Class becomes a community of mathematicians deciding together on what is true.
- Students move beyond “Level 0” — *the book says so; my teacher told me.*
- This approach requires a shift as students start viewing themselves as mathematical thinkers and doers.



# Math argumentation requires new norms





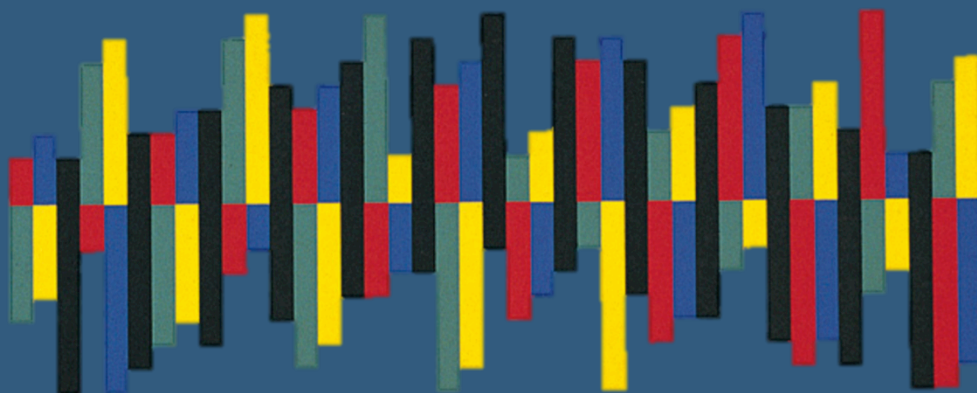
## Challenge:

How do we engage students whose home and prior school norms may not be perceived as aligning with norms for mathematical argumentation?

# Insight: engaging in mathematical argumentation is improvisational!

Effective classroom discussion is *improvisational*, because the flow of the class is unpredictable and emerges from the actions of all participants, both teachers and students.

—Dr. Keith Sawyer, 2004





# Teachers use improv games with students.

- Games come from improvisational theater
- Rules structure freedom to participate
- Freestyling as improv

In movies



Lin-Manuel Miranda at the White House



# Qualities of improv games

- Provide high-energy modalities
- Foster sense of belonging
- Kinetically rich experiences bridge everyday experiences and academic content
- Make norms explicit for argumentation

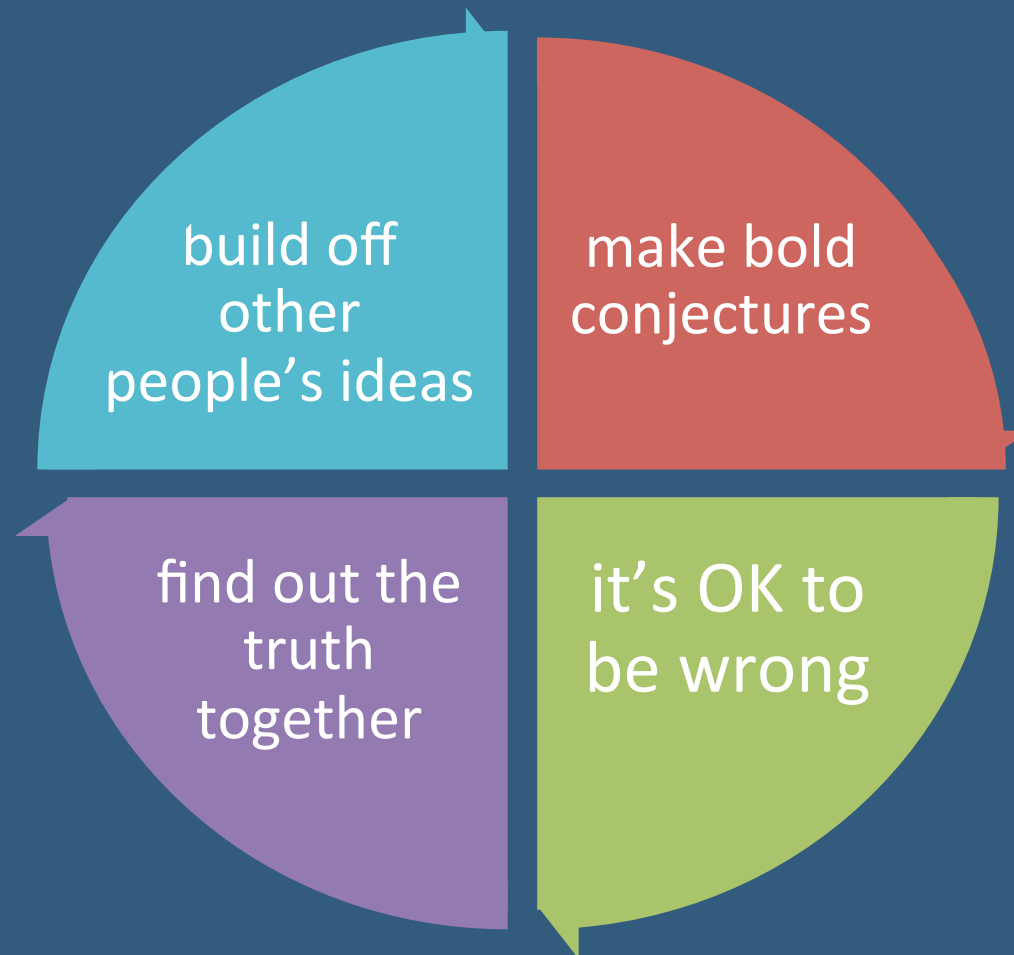
# Let's play: *Gift Giving*



# Let's play: *Gift Giving*

- Stand facing your partner.
- There's a huge closet of unlimited gifts behind you.
- One person is the **giver**, another person is the **receiver**.
- The **giver** offers the receiver a wrapped gift from the closet.
- The **receiver** opens the gift and gratefully describes (and names) the gift.
- The **giver** responds with an explanation of how and why the gift was selected and why it would be enjoyed.
- Then switch roles.

# What does the game have to do with norms for argumentation?





Improv games . . .

are used for making classroom norms explicit for argumentation

# Teacher M's students connected the game to argumentation norms.

Because like, if discussion takes time, one person's doing at a time and nobody's talking over each other.

It connects because when we were doing zip, zap, zop, we were going back and forth and it's like speaking, but actually a game. And when we were doing the argument we are also doing the same with ideas instead of using zip zap zop.

When we were making our conjectures, it's okay if we are not right the first time as long as you try.

We was speaking loudly enough so everyone can hear.



# Curriculum



Paper-and-pencil and online instructional activities can enrich the learning environment for culturally and linguistically diverse students engaged in argumentation.





## Task: Operations on Signed Numbers

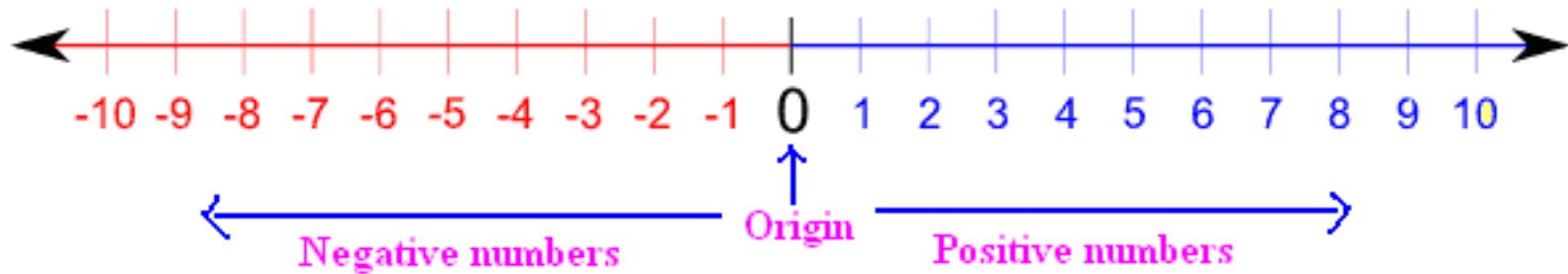
**You know how to add signed numbers using the number line.**

**Find the sums of different combinations of positive and negative numbers and notice whether the sum is positive or negative.**

1. What patterns do you see?
2. Make conjectures about a shortcut for adding signed numbers without using the number line.
3. Justify one conjecture based on what you know about signed numbers and addition. (Remember, your conjecture may turn out to be true or false.)
4. Write down your conclusion.



# Adding Signed Numbers on the Number Line





# Online curriculum

Students do not work in isolation:

- Students work in groups with individual and collective accountability.
- Teacher interaction is required.
- Online prompts are paired with interactive multiple representations.

# Dynamically linked multiple representations and prompts provide opportunities for discourse and gesture

The interface displays a simulation of a red robot named 'Shakey' on a track. The track is marked with distance in centimeters (0 to 40). The robot is currently at 8.00 cm. The simulation is paused at 00.0 seconds.

**Position Graph**

The graph shows Distance (cm) on the y-axis (0 to 40) and Time (seconds) on the x-axis (0 to 11). A red line starts at (0, 8) and passes through (4, 24).

Time (seconds)	Distance (cm)
00.0	8.00
01.0	12.00
02.0	16.00
03.0	20.00
04.0	24.00

**Data Table**

Time (seconds)	Distance (cm)
00.0	8.00
01.0	12.00
02.0	16.00
03.0	20.00
04.0	24.00

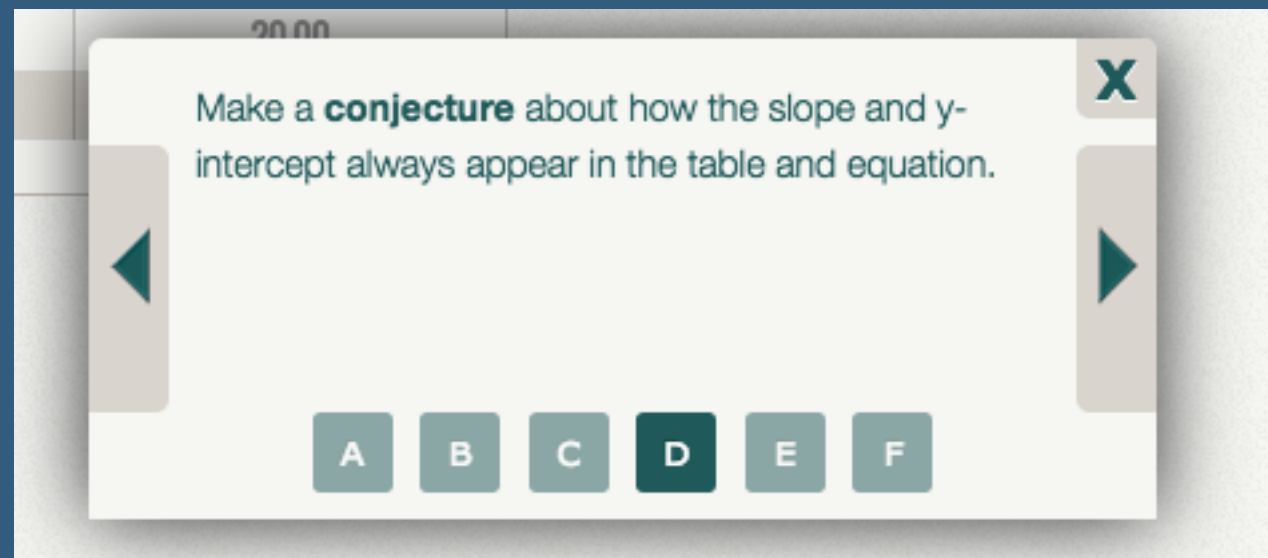
**Function**

$$y = 4.00x + 8.00$$

**Prompt:** Create two new robot trips that do not start at zero. For each trip, how are the slope and y-intercept represented in the table and equation?

Answer options: A, B, C, D, E, F

# Vocabulary specific to argumentation and mathematics content



20.00

Make a **conjecture** about how the slope and y-intercept always appear in the table and equation.

◀ ▶

A B C **D** E F

X

The image shows a digital interface for a math problem. At the top, there is a timer showing '20.00'. The main text asks the user to 'Make a **conjecture** about how the slope and y-intercept always appear in the table and equation.' Below the text are two large, light-colored buttons with dark green triangles pointing left and right. At the bottom, there are six small, light-colored buttons labeled 'A', 'B', 'C', 'D', 'E', and 'F'. The 'D' button is highlighted with a dark green background. In the top right corner of the interface, there is a small dark green 'X' button.

# Teacher notes provide specific moves that challenge students to engage in argumentation.

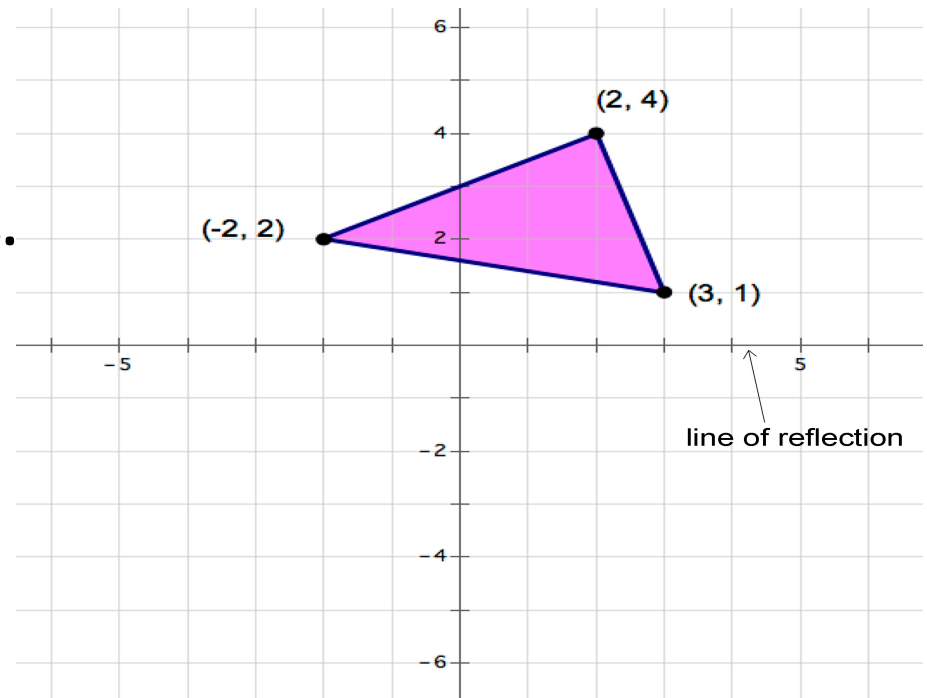
## Conjecturing

D.  
Make a **conjecture** about how the slope and y-intercept always appear in the table and equation.

This conjecture should go beyond specific quantities involved. Challenge students to make generalized statements by prompting them to look for patterns across cases that they explored. For example, if students said "increase by 2" in the previous activity, they might say here "increase by a constant" by recognizing the pattern across cases they created. Challenge students by asking, Would the tables all have to have  $x$  increase by 1? What would happen if that wasn't the case?

# Task: Triangle Reflection

Reflect the triangle over the  $x$ -axis.



1. What patterns do you see in the coordinates of the vertices of the triangle and its image?
2. Write a conjecture that describes what happens to the coordinates of the vertices of any triangle reflected over the  $x$ -axis.
3. Justify your conjecture.
4. Write down your conclusion.

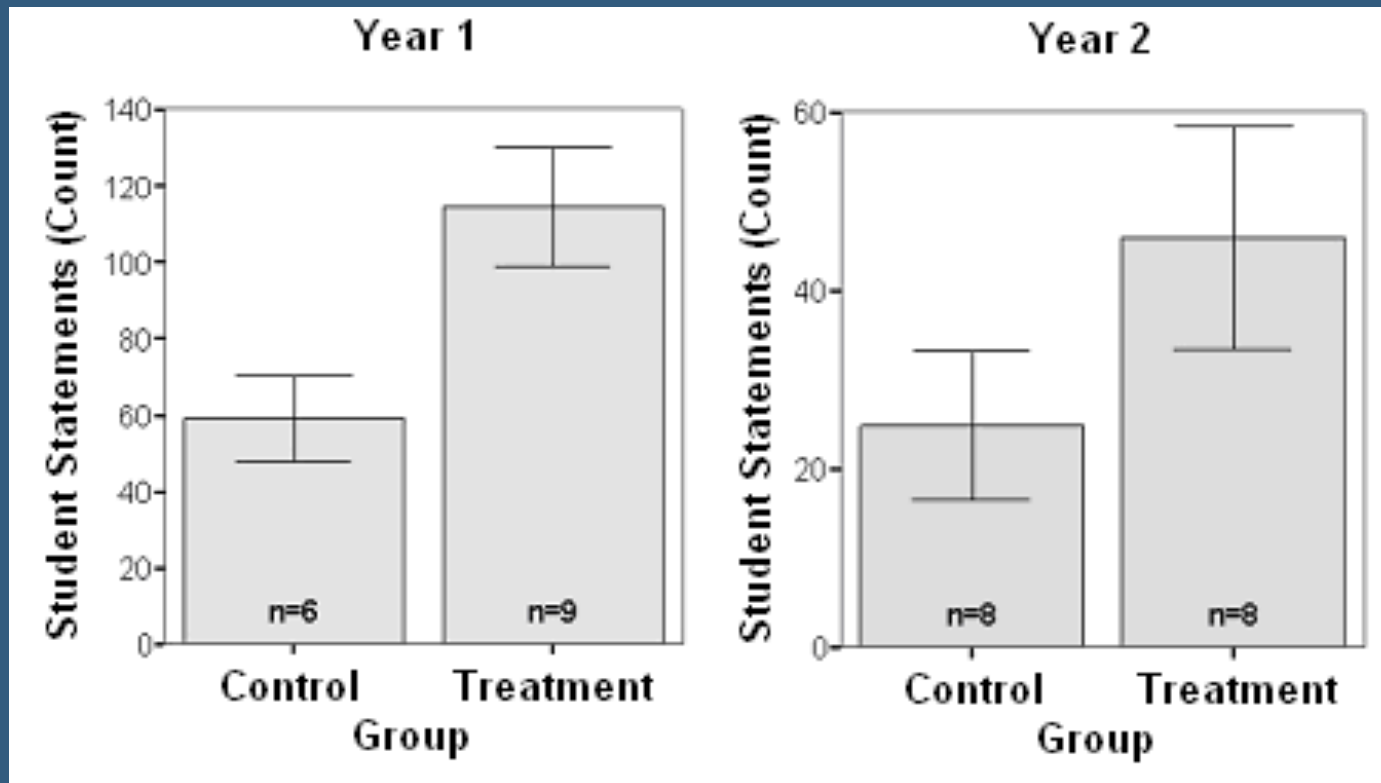
# Evidence of effectiveness

In an impact study, Bridging students engaged in twice as much argumentation as “control” students.

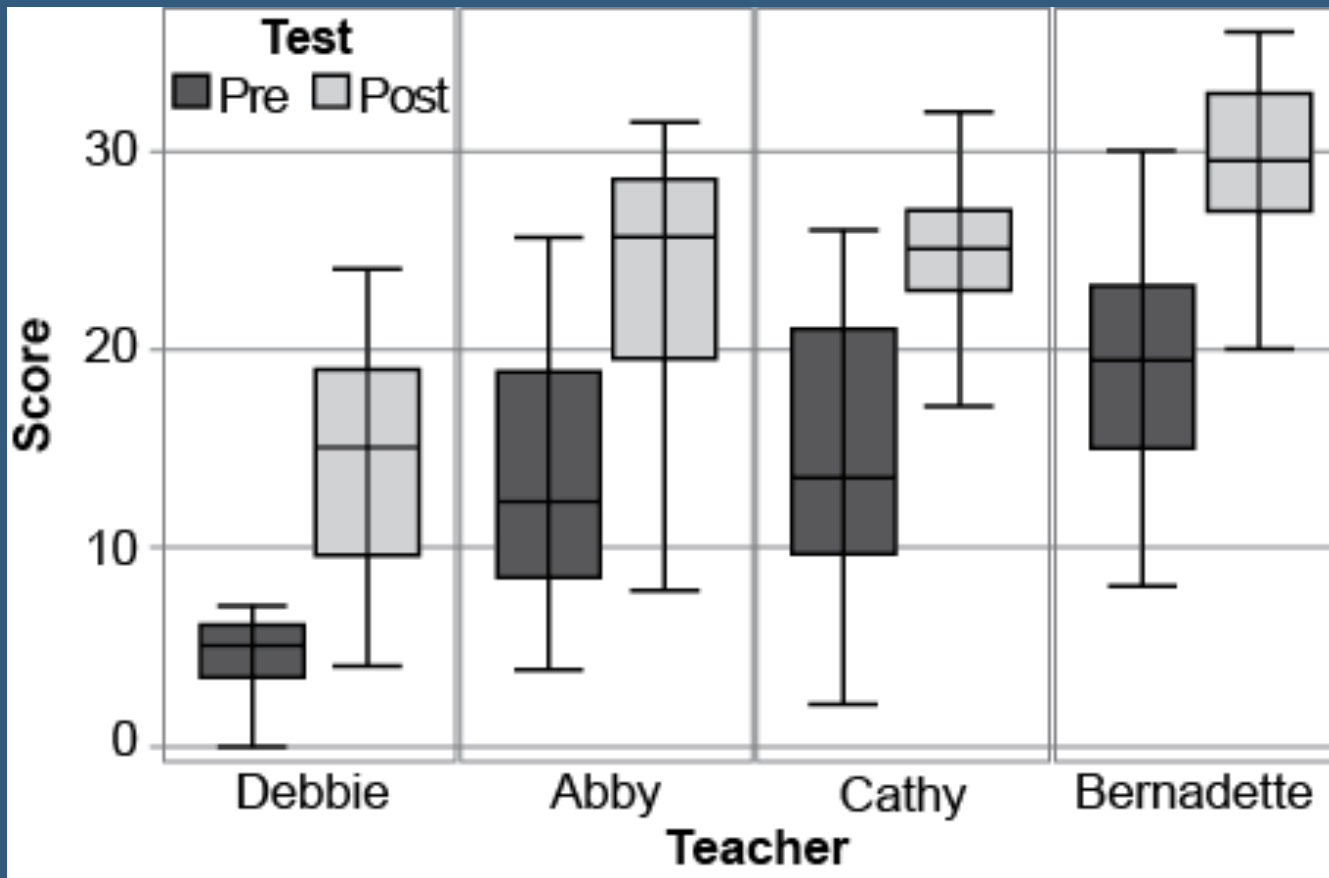
In a study of four diverse classrooms, students learned both content and argumentation skills in discourse and writing.



# Study 1: Results show twice as much classroom argumentation



## Study 2: Results show a gain of 10 pts in student learning





# Discussion

- In your view, how do these activities address the student needs we outlined?
- What is your own experience in supporting the needs of these student groups?
- What other needs should we consider?



# Online Resources Demo

[Bridgingmath.com](http://Bridgingmath.com)

New book for teachers based on this work.

Coming  
September,  
2017

Jennifer Knudsen  
Harriette S. Stevens  
Teresa Lara-Meloy  
Hee-Joon Kim  
Nicole Shechtman

# Mathematical Argumentation in Middle School

The What, Why, and How

A Step-by-Step Guide with  
Activities, Games, and  
Lesson Planning Tools





# One more presentation

Session 696

Learning to Teach Mathematical Argumentation  
through Successive Approximations of Practice

Saturday 11:30am-12:00pm



THANK YOU!