A Professional Development Program for Mathematical Argumentation:

Bridging from Workshop to Classroom

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Bridging
professional development

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Contents

Imagine a seventh-grade classroom where the students are discussing whether or not the placement of quadrants in a coordinate system is a convention, and the teacher is adeptly prompting them to question and expand on each other's arguments by asking them to clarify and refine their conjectures, and by stepping aside to let them make their own arguments. This engagement is the result of an innovative professional development (PD) program that supports middle school teachers in teaching for mathematical argumentation.

Mathematical argumentation is the foundation not only for developing deep conceptual understanding of math, but also of making logical connections among abstract ideas. Teaching it to middle-schoolers is thus essential as a prerequisite to their pursuit of higher math and as a life skill. Yet some teachers themselves are not proficient in mathematical argumentation. The challenge is to equip teachers with knowledge and techniques so they feel confident and are effective in teaching argumentation.

Teachers often find that they cannot apply what they learn in PD workshops because it does not fit the realities of the classroom. Our intent was to create PD that provides bridges from workshop learning to the classroom. This Bridging PD model has produced improvements in classroom argumentation and gains in student learning.

Definition and Importance of Mathematical Argumentation

The widespread adoption of the Common Core State Standards for Mathematics (CCSSM) (National Governor's Association, 2010) has created the need for PD that helps teachers implement the eight mathematical practices, one being argumentation. "Constructing viable arguments and critiquing the reasoning of others" require a rich set of oral and written practices, both mathematical and interpersonal or social. These practices include making conjectures and using logic to explore their truth, recognizing and using counterexamples, and pointing out and explaining flaws in arguments. Using interpersonal or social practices, mathematically proficient students communicate their arguments to others and listen to others, asking questions to decide whether arguments make sense.

Mathematical argumentation has many definitions. We developed one that would both support our teachers and represent research drawn from various perspectives. We started with the oft-cited structure of argument as laid out in rhetorical theory by Toulmin (1958, 2003). In this scheme, an argument in any discipline consists of core elements (i.e., a claim about something, data as evidence to support the claim, and warrants that explain the relationship of the data to the claim). To make a scheme appropriate to mathematics and the needs of the mathematics teacher, we created a three-step structure of conjecturing, justifying, and concluding. On the basis of our view of mathematical argumentation as a social process, we defined ***conjecturing*** as a process of "conscious guessing" (Lakatos, 1976) to create mathematical statements, or claims, of as-yet-undetermined mathematical validity and defined ***justifying*** as a process of explicating one's reasoning, using data and warrants, to establish the mathematical validity of a conjecture. We characterized concluding as the process of coming to consensus or agreement about the validity of the conjecture and its justification (Lampert, Rittenhouse, & Crumbaugh, 1996; Thurston, 1998). These definitions enabled us to identify argumentation in classroom observations, and the three categories served as a framework for teachers to use in learning and teaching argumentation.

It's not just the CCSSM that places a priority on mathematical argumentation. Mathematics educators, mathematicians, and philosophers agree that argumentation practices are fundamental and essential for learning mathematics (Kuhn, 2005; Lakatos, 1976; Lampert, 1990; Romberg, Carpenter, & Kwako, 2005; Stylianou, Blanton, & Knuth, 2009; Thurston, 1998; Yackel, 2001). The National Council of Teachers of Mathematics' Principles and Standards (2000) and the National Research Council's ***Adding It Up*** (2001) also assert that argumentation is an important part of students' mathematical education. Argumentation provides students the opportunity to engage in authentic mathematics. It enables them to take on mathematical authority and intellectual autonomy—foundational building blocks for developing deep conceptual understanding. Argumentation is an important 21st century workplace skill. Making logical connections among abstract ideas and interacting with others to clarify their ideas are both necessary in an increasing number of good jobs (Partnership for 21st Century Skills, 2008).

Even though argumentation is important for many reasons, we found few PD programs with a central focus on it. This led us to create such a program.

Design Principles for the Bridging PD Model

We conceptualized Bridging PD as a way to address the phenomenon that most PD providers have encountered at one time or another: seeing little change in teaching despite best efforts in PD workshops. It may be that these PD models lacked adequate connections to actual practice. Thus, the Bridging model is based on the idea that PD should provide bridges to classroom practice so teachers can transfer what they learn in PD to the classroom. By bridges, we mean PD activities that are close to the experience of actual teaching. In related research, Grossman and colleagues (2009) identified "representations and approximations of practice." For example, in reading an article teachers may acquire important information about the practice of argumentation, but the act of reading is far from the act of teaching; so reading can provide a representation of practice, but it is not a very near approximation of practice. We sought to design PD activities with more authenticity relative to classroom realities: near approximations of practice along with good representations of practice. We conjectured that these bridging activities would be important in helping teachers take newly learned teaching moves back to the classroom and actually use them. This core idea led to a set of design principles. As we describe each, we'll explain how they were applied in our PD.

Mathematical knowledge for teaching is foundational in learning to teach for argumentation. Mathematical knowledge for teaching is a broad term used to include, not exclusively, both general and specialized knowledge of mathematics (Ball, 2008). Of particular importance for Bridging is what Ball and colleagues called ***specialized content knowledge for teaching***, including examining alternative representations, providing explanations (for rules), and evaluating unconventional student methods. We posit that the specialized knowledge unique to argumentation includes the use of generalization or counterexamples to advance or disprove students' arguments, knowing why certain rules worked as legitimate justification, and being able to highlight the mathematical logic in students' statements, such as being able to recast students' arguments as "If…then" statements.

We aimed for teachers to develop knowledge that is specific to the curriculum they teach and that is appropriate to beginning argumentation practices. We hypothesized that teachers would have diverse mathematical needs when coming to Bridging PD and that these needs would be related to both argumentation skills and mathematics content. Few teachers at the middle school level are likely to have made mathematical arguments themselves. Many would need to learn the mathematics that would be the context of argumentation, whereas others would be more prepared with this knowledge. Indeed, in the implementation of the Bridging PD, we found that some teachers did need to learn the vocabulary and conventions of the coordinate plane while others were ready for more advanced work in finding, say, the relationship between the equations of perpendicular lines. In this implementation, we supported teachers' developing understanding of the role of counterexample in classroom discourse. Our research results bore out our hypotheses in that we found a strong correlation between teachers' understanding of general and specialized content knowledge and the length of the mathematical arguments they were able to support.

Curriculum aids in supporting classroom argumentation. Curriculum materials can be an important source of new ideas about how and what to teach. Cohen and Hill's study (1998) on California reform showed that when grounded in student curriculum, teachers' learning affected their practices and students' learning. Cohen and Ball (1999) also pointed out the importance of "specification" of the curriculum for effective implementation. They suggested developing targeted units to replace specific topics in the mathematics scope and sequence, such as rate or proportional reasoning. Teaching with these so-called "replacement" units has been shown to have an effect on student learning (Roschelle et al., 2010). This approach is leveraged in Bridging.

In our work, three different sequences of curricular activities, ranging from 4 to 10 lessons, support teachers in introducing mathematical argumentation into their classroom. The sequences support teachers several ways. First, we use the curriculum as a learning tool for teachers—a way for them to learn new content and also as a support for new teaching moves. Then back in the classroom, the curriculum unit can provide coherent, structured experiences for students and teachers that scaffold argumentation about specific content, and the unit can also provide prompts and questions. The development of teaching moves, discussed next, builds on the foundation of knowledge teachers build through their experience with the curriculum.

For example, one of our units is about rectangles in coordinate geometry. Students are asked to find patterns of coordinates in rectangles, make conjectures based on these patterns, and then justify them. First, students learn the conventions of coordinate geometry. Then they plot the vertices of four rectangles, looking for and articulating patterns in the coordinates of the vertices. They then make generalized conjectures about the patterns and justify them. The task is open ended enough that very simple or more sophisticated conjectures can be made and argued for. Students' conjectures could be as simple as, "For any vertical line in the rectangle, the *x*-coordinates are the same." More sophisticated conjectures employ the use of variables: "If working with rectangles parallel to the axes, then the coordinates of vertices will be in the pattern (*a*,*c*), (*b*,*c*), (*b*,*d*), (*a*,*d*)."

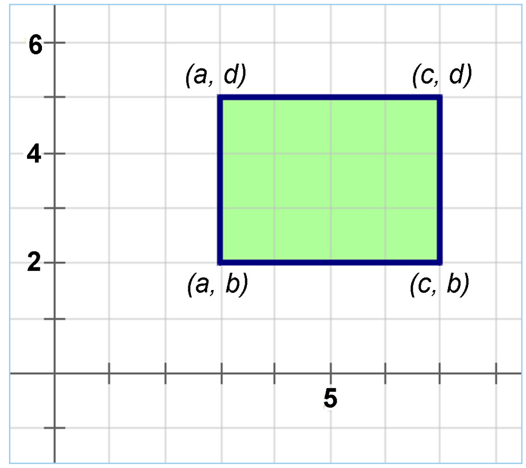

Figure 1. Coordinates of the vertices of a rectangle.

Thus, curriculum serves as a representation of practice in both the classroom and the PD workshop. It brings the structure of argumentation from the workshop to the classroom and provides opportunities for teachers to use moves, discussed next, that support stages of conjecturing, justifying, and concluding.

Teaching is improvisational, and teaching moves can be taught through improvisation games. Although teaching can have its scripted or routine aspects, we chose to focus on its improvisational nature. This is particularly important when a teacher is guiding students from novel conjectures to previously unknown conclusions and when students must enact interpersonal practices of argumentation while constructing arguments together. The flow of classroom discourse unfolds unpredictably and emerges dynamically from the actions of the teacher and students working together Borko & Livingston, 1989; Lobman, 2007; Sawyer, 2011)—through "collaborative emergence" (Sawyer, 2003). We looked, then, to improvisational theater for inspiration in how to help teachers develop the in-the-moment moves they would need to support students in the act of argumentation.

In improvisational theater, actors work together to develop dramatic scenes extemporaneously in front of an audience. Improvisation thus is a form of collaboration, creativity, and communication as a performance art (Madson, 2005). Actors learn to do improv theater by engaging in a wide range of "improv games" that enable them to develop the ability to co-construct characters, relationships, pantomimed objects in space, and interesting plotlines. Improv games have simple rules of interaction to follow that foster spontaneity, strong collaboration, and playfulness among participants. The rules govern variation across characters, setting, and central problems to solve so that a wide variety of scenes can be played out. "Applied improv" is the adaptation and use of improv games in contexts other than acting, such as health care, wellness, therapy, training, communication, soft skills, and K–12 education (e.g., see http://www.appliedimprov.com/).

Figure 2. Improv games help learn norms for argumentation.

We used improv games in our PD to establish new kinds of norms for interaction that could ultimately be taken back to the classroom. These norms emphasized playfulness, full participation, making bold conjectures that may or may not turn out to be true, and building off of each other's mathematical ideas as an ensemble. Other games emphasized paying close attention, spontaneity, building narratives together, and having the freedom to try out justifications that may turn out to be wrong. All these games were designed to mitigate against the norms for quick and correct responses to routine questions that are typical in many mathematics classrooms.

We also designed a new set of games we called "teaching games." These games are intended to help teachers focus on and practice specific teaching moves that they would use improvisationally to support argumentation in the classroom. For example, questioning is an important part of supporting argumentation, and we wanted teachers to see the specific roles that both open- and closed-ended questions could take. So we designed a game to that end. Teachers play in groups of four. One teacher is assigned the role of teacher, and the other teachers play students. A specific argument is chosen as the context for the teaching. The game is played in two rounds of about 15 minutes each. In the first round, the teacher must facilitate the ongoing argument only using close-ended questions and by giving information. In the second round, the teacher must ask only open-ended questions. At the conclusion of the game, teachers discuss and compare their use of these questions, highlighting what each is good for. The idea is not to persuade teachers of the superiority of one type of question over another but to provide vivid experience in using those questions and reflection on the purposes to which they could be put.

A series of teaching games leads up to a culminating performance of a slice of classroom action. In this activity, a teacher chooses part of a lesson to enact, enlisting her fellow teachers to act as students with specific characteristics. For example, a teacher may want to try out concluding an argument with the whole class because this tends to be a challenging part of argumentation. She can assign roles to one teacher as the student who always wants to give the right answer and to another as the student who is reluctant to speak up in class. The group then plays out the scene, with a facilitator who can call time out to provide commentary or lead group decision making about what moves to make next. As a close approximation of practice, this activity provides teachers with an opportunity to put together newly learned moves to support argumentation but still allows for reflection on the episode to maximize the learning opportunities in the activity.

As an example of how improv teaching games served as a bridge to practice, an early version of a teaching game helped teachers explore the depths and limits of the question why. In the game, teachers were asked to repeatedly ask versions of why questions until uncovering students' most basic ideas about the content, their informal axioms. In one Bridging teacher's classroom, Esther[1] took up the practice of asking why in a series of dialogues with students that took place in front of the whole class. Through repeated use of variations on why, Esther helped students move from arguments based on mathematical procedures to arguments based on mathematical concepts to justify why rectangles in a set were similar. Students often began with procedural arguments for why their triangles were similar: basically, "I made them that way." Esther's questions probed until the multiplicative relationship between corresponding sides was revealed.

1. All teachers' names are pseudonyms.

Improvisational teaching games are a core activity in the Bridging PD. They enable teachers to experience, in a safe environment, a slice of teaching practice in action. The activity in them is authentically one of teaching, through carefully bounded rules that help teachers try out new moves and reflect on their purposes. These rules are what move teaching games beyond simple role-play. They focus the action on particular practices and parts of a lesson.

Planning is the complement to improvisation. In studies of teacher planning, Clark (1988) found that many teachers do not make written plans but do most of their planning mentally. Studies of expert and novice teachers (Borko & Livingston, 1989) indicated that expert teachers tended to make extensive use of mental plans instead of relying on written plans because they have well-organized knowledge of pedagogy. Sawyer (2004) related planning and improv through what he called “disciplined improvisation”—referring to the studies on expert teachers who use routines and activity structures in a flexible and improvisational manner. Outside education, in research on innovation, planning is seen as going hand in hand with improvisation, where each contributes to an important new product or process (Sawyer, 2004, 2008).

In Bridging, improv games are used in tandem with careful planning in which teachers decide when specific teaching moves they are likely to use. The kind of planning we promote is not the production of conventional lesson plans. Instead, the plan is a map on which the improvised teaching route is based. We use teachers’ inclinations to do mostly mental planning to help them create written plans through think-aloud. We ask them to visualize what they would be doing and saying during a lesson and what students would be likely to do and say as a result. They think aloud as they visualize, and a partner teacher takes notes. Then, teachers turn these visualizations into detailed plans from which they can rehearse their teaching. Planning provides teachers with self-generated representations of practice which can then guide their participation in approximations of practice.

Argumentation for students is improvisational and can be learned through improvisational games. Bridging addresses both the social and sociomathematical norms that support these interpersonal practices. Social norms include students actively listening to one another and treating each other with respect so that they can feel comfortable risking making mistakes and critiquing one another’s ideas (Boaler, 2002; Boaler & Staples, 2008; Wood, 1999; Yackel, 2002). Sociomathematical norms for argumentation include expectations that students explain justifications and critique the reasoning of others and an understanding of what counts as a suitable justification (Wood, 1999; Yackel & Cobb, 1996).

How to Do Math Argumentation

Conjecturing

- Look for math patterns that make sense to you.
- Think about more than just one case.
- Be creative.
- Don’t judge other people’s conjectures.

Justifying

- Look for reasons why a conjecture is true or false.
- Consider examples and counterexamples.
- Build off of other people’s ideas.
- Generalize.
- Try to convince others of your ideas, but keep in mind that you could be wrong…which is OK.

Concluding

- End an argument when your class is convinced that the conjecture is true (or false).

Figure 3. This poster makes explicit the norms and behaviors expected from students during each of the stages of argumentation.

We provide teachers with a set of tools and rules that they can use in the classroom to establish and maintain the appropriate social and sociomathematical norms. Establishing norms is important for all students but particularly those for whom argumentation may be a foreign practice (Lubienski, 2007). One set of tools for developing and sustaining norms for argumentation is games that students play in class, either in a whole group or small group. These games, like the games for teachers, are based on applied improv games.

For example, the *School Trip Planning* game is intended to help students learn to build off each other's ideas. Pairs of students come to the front of the room to plan a school trip, in two rounds. One student is given the role of idea generator in each round. The idea generator's job is to suggest new ideas to be used in planning the trip. The other student then responds as he or she helps plan. In the first round, each response to new ideas must be a sentence beginning with the word no. In the second round, the response to new ideas is "Yes, and...." Students can note important differences between the two rounds of the game. In the first, little progress is made as competing ideas are aired but never built on. In the second, the responder can still include his or her own ideas, but the ideas are incorporated into a single plan. "Yes, and…" is a basic rule of improv and corresponds quite directly to the mathematical argumentation behavioral norm, "Build off of other people's ideas," which is at the heart of the social process of argumentation. Through discussion, teachers can help students make the connection between the game and argumentation.

Tools and rules are used in both PD and the classroom. As representations of practice, they represent a set of social and sociomathematical norms that structure teachers' and students' experiences when they are engaged in argumentation; they carry with them the teaching moves established in the workshop.

One of the teachers, Stephanie, showed us the utility of tools for argumentation. We had made a poster for the PD workshop to quickly communicate the conjecturing, justifying, and concluding steps of argumentation. Stephanie appropriated it for use in the classroom, and she structured her classroom discussion by pointing to the poster to remind students of the step of argumentation they were supposed to be participating in. She strictly separated the acts of making conjectures and justifying them. She also imposed rules derived from the "Yes, and…" improv rule, basically enforcing a rule that no judging of conjectures was allowed during the conjecturing phase. This gave students the freedom to come up with inventive conjectures. Stephanie did not merely bring an artifact back to her class; that artifact carried with it a set of teaching moves that supported argumentation. Her use of the poster was similar to the ways the poster had been used in the PD workshop but adapted for her own rather strict but effective teaching style. Her teaching inspired us to create posters with lists of productive argumentation behaviors that could be used by other teachers.

Implementation of the Bridging Model

We have used Bridging PD in two configurations. In *Bridging I*, our first iteration of the program, PD was conducted over a 2-week period, and offered to two different cohorts. Participants first spent five 6-hour days on mathematical knowledge for teaching (MKT), using the Bridging curriculum unit they'd be using with students but with appropriate modifications for adult learners to deepen the specialized mathematical knowledge necessary for teaching. The first cohorts' unit was on similarity and the second's on coordinate geometry. The next 4 days of the workshop were on pedagogy. This is where we used the improvisational teaching games and planning through visualization to help teachers develop a small set of new teaching moves and get ideas for when and how to use them.

The *Bridging II* PD consisted of four 5-hour Saturday sessions, a 3-day workshop, and 1 planning day. Four co-designer participants were from schools around the San Francisco Bay Area in communities of different socioeconomic backgrounds and student achievement. In the Saturday sessions, the teachers were introduced to argumentation and participated in the co-design of a 2-week replacement unit on the geometry of triangles. The unit included a set of tools and rules for establishing classroom norms. In the 3-day workshop, we used a mix of training on implementation of the replacement unit, learning to use posters and improv games with students, improv teaching games, and planning sessions. Finally, a planning day enabled teachers to weave these components together, using visualization and written plans.

Research on the Bridging Model

Integral to the design and development of the Bridging model was research on impacts in the classroom. For each configuration of the program, we conducted studies that examined the discourse that occurred in the teachers' classroom. Here we discuss how we studied effects on teacher knowledge, student knowledge, and students' opportunities for doing mathematical argumentation.

Bridging I. We designed Bridging I to answer the question: ***Through participating in the Bridging PD program, do teachers grow in their MKT and have increased argumentative talk in their classrooms?*** The program was implemented in a 2-year small-scale randomized impact study. With a particular focus on teachers of underserved student populations, we recruited middle school math teachers from high-poverty urban districts in the San Francisco Bay Area. Each year was a stand-alone program—year 1 addressed argumentation with the content of similarity, and year 2 addressed argumentation with the content of coordinate geometry. Most teachers attended for either year 1 or year 2 only, although a few attended for both years.

All teachers were randomly assigned to a treatment group or a control group. Teachers in the treatment group had the full Bridging PD—MKT in the first week and pedagogy in the second. Teachers in the control group had the MKT in the first week, but in the second week attended a different PD session of equal value to them but targeting different pedagogical knowledge, vertical coordination across grade levels. This experimental manipulation enabled us to examine the impacts of the first week's MKT PD alone versus the MKT PD ***coupled with*** the PD on teaching moves and bridges to classroom practice.

1. There were two main outcome measures for all teachers in both groups:
2. We administered a pre-workshop and post-workshop MKT assessment aligned with the content of the MKT in the workshop.

During the school year, we observed teachers in both groups for 2 days in their classrooms teaching with materials covered in the first week of the PD. In these observations, we videotaped whole-class discussion and transcribed the teacher and student talk verbatim.

Our analyses focused on students' argumentative talk and teachers' teaching moves to support argumentation.

We found several positive results. The changes in teachers' scores on the MKT assessment between the beginning and end of each workshop indicated that teachers in both the treatment and control groups grew in their understanding of the mathematics. In year 1, the focus was on argumentation about similarity, and there was an effect size of .5 [$t(23) = 4.2$, $p < .001$];in year 2 the focus was on argumentation about coordinate geometry, and there was an effect size of .3 [$t(20) = 4.5$, $p < .0001$].

The argumentative discourse in treatment classrooms was approximately double that in control classrooms in both years 1 and 2 (Figure 4). Argumentative talk was measured by the number of student turns of mathematical talk in whole-class discussion in which students were constructing justifications of conjectures. In year 1, this difference was statistically significant with an effect size of 1.4 [$t(13) = 2.6$, $p < .05$], whereas in year 2, this difference was a marginally significant trend with an effect size of .7 [$t(14) = 1.4$, $p = .18$].

In year 1, treatment teachers did more of the teaching moves that were coded as supporting argumentation: closed-ended questions [$t(11.5) = 2.2$, $p <.05$], open-ended questions [$t(12.0) = 2.8$, $p <.05$], and moves to facilitate and encourage participation by multiple students [$t(11.8) = 2.4$, $p <.05$]. In year 2, there were nonsignificant trends indicating that treatment teachers did more of each of the examined teaching moves.

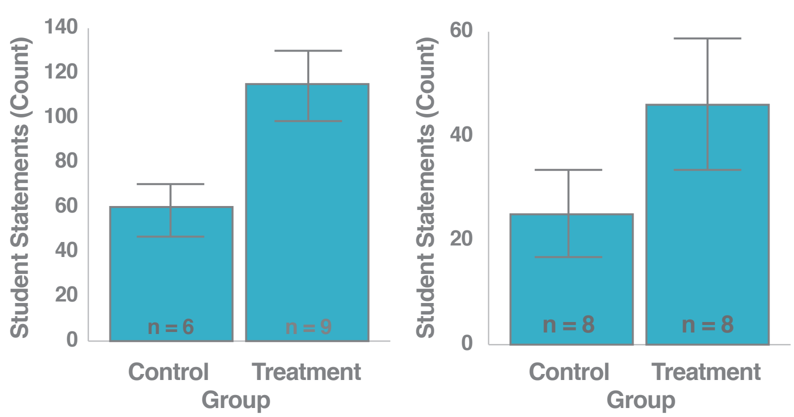

Figure 4. Argumentative Talk: Mean Counts (± standard error) of Student Statements

Note: The number of statements is not comparable across years; the curriculum materials were different.

Although these main effects were substantial, significant variation also existed between teachers in how much argumentation occurred in each classroom. As designers, we wanted to know whether there were any patterns that might indicate consistent reasons why teachers might underperform and whether we might be able to design additional supports for the teachers. Therefore, we examined potential mediating variables that might contribute to less classroom argumentation. The most important mediator was teachers' prior MKT. In both years, middle school math teachers came to our PD with a wide range of prior knowledge—on both years' pretests, teachers' scores spanned the full range, from less than 15% correct to 85% and higher. This had important consequences in the classroom. Whereas treatment teachers overall had more argumentative talk in their classrooms, the average length of the arguments was highly related to their content knowledge.

Our work also revealed that the interpersonal practices of mathematical argumentation are novel for many teachers and that teaching for argumentation was challenging regardless of prior specialized content knowledge for teaching. In the PD, teachers expressed uncertainty and skepticism about using what they perceived to be more open-ended practices in their classrooms. It became apparent in PD discussions that they needed support in setting up their classroom culture so that argumentation was a possibility.

Bridging II. In Bridging II, our core research question was: ***How can we extend and expand the Bridging PD model to enhance students' opportunity to engage in argumentation and learn mathematical content?*** We extended the program in four ways:

1. Developed a new 2-week replacement unit, Argumentation with Triangles (on geometry content), with more explicit supports for teachers with weak MKT
2. Added a set of tools that teachers could use to establish classroom norms and structure interpersonal practices for argumentation
3. Provided richer opportunities for students to engage in argumentation through group work, written argumentation, and using dynamic representational software (Geometer's Sketchpad)
4. Examined students' understanding of content and competency in argumentation through assessments and interviews.

Our design research examined four teachers' implementation of the unit and what their students learned. Implemented in fall 2012, the teachers taught the Triangles unit at the beginning of the school year so it could be used to establish classroom social and sociomathematical norms. We asked each teacher to select a target class that we would observe most days of the unit. We observed a range of classes, from an advanced geometry class to regular seventh-grade math to a class that supported the needs of students with learning disabilities. An assessment was administered to the students in the target class before and after the unit. We used the pretest and posttest to identify students with a range of ability in written mathematical argumentation and conducted follow-up interviews to examine their ability to make spoken arguments.

In all the target classrooms, students demonstrated substantial learning gains, with an effect size of 1.45 [$t(96) = 9.94$, $p < .0001$], on average gaining 10.31 points out of 36 from pretest to posttest (Figure 5).

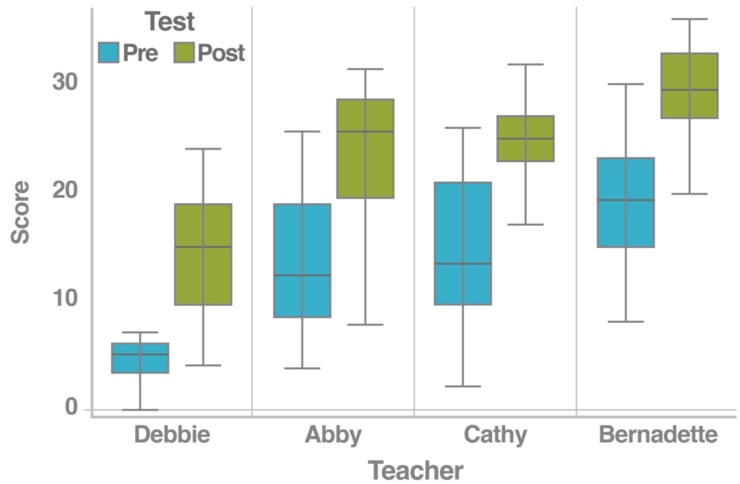

Figure 5. Student Learning Results for the Bridging Triangle Unit

We also used this design study to continue to identify challenges for practice and opportunities for improvement. Not surprisingly, our findings from the interviews indicated that after two weeks of instruction, students were successful in basic argumentation, but there was still room for growth in argumentation skills. This points to the need for a longer, more comprehensively integrated PD program. Additionally, in a focus group, project teachers discussed challenges they faced with struggling students, providing important insights into the learning needs of a variety of students.

Conclusion

Creating professional development with measurable change in classroom practice is challenging. Bridging met that challenge. Teachers who participated in the Bridging PD had measurable increases in the teaching moves that support argumentation and student argumentative talk in their classrooms. With instructional units on the mathematical practice of argumentation and professional development that supported it, students made mathematical arguments while learning content.

The five design principles for our approach are essential elements for teachers taking on new complex practices. Teachers need a combination of stable routines and the ability to respond improvisationally in the moment to student ideas. They need a strong understanding of the mathematics and how to do argumentation, they need curriculum that provides opportunities and supports for doing argumentation, and they need to be able to plan for argumentation while also having the flexibility to improvise with students as they generate spontaneous mathematical talk.

There is increased interest in professional development that prepares teachers for structured spontaneity in the classroom. We offer the Bridging model as an approach to support those kinds of interactions, which are increasingly important as we aim for student understanding of content and authentic participation in mathematical practices.

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